Hydromechanika a hydrológia

Prof.Ing.Karel Kovářík, PhD
Katedra geotechniky
Stavebná fakulta ŽU

Tel. 0905-659763
E-mail: kovarik@fstav.uniza.sk

Základné vlastnosti tekutín

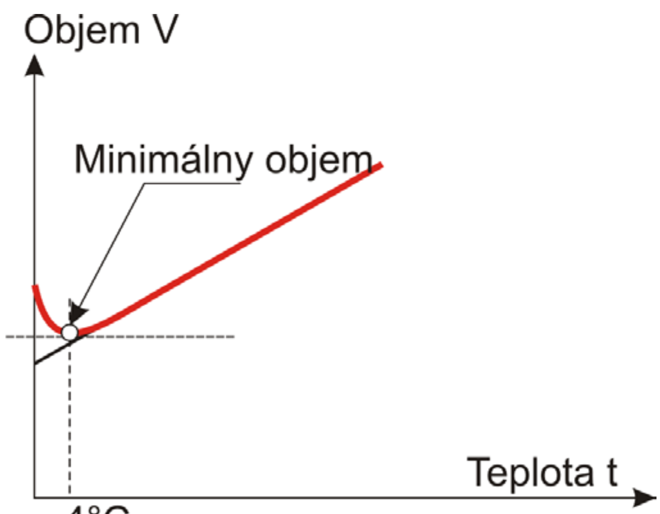
- Hustota $$\rho = \frac{m}{V}$$

- Objemová teplotná rozťažnosť

$$\alpha = \frac{1}{V_0} . \frac{\Delta V}{\Delta T} \Rightarrow V = V_0 . (1 + \alpha . \Delta T)$$

α – koeficient teplotnej rozťažnosti

Anomália vody – maximálna hustota pri 4°C



Základné vlastnosti tekutín

- Stlačiteľnosť

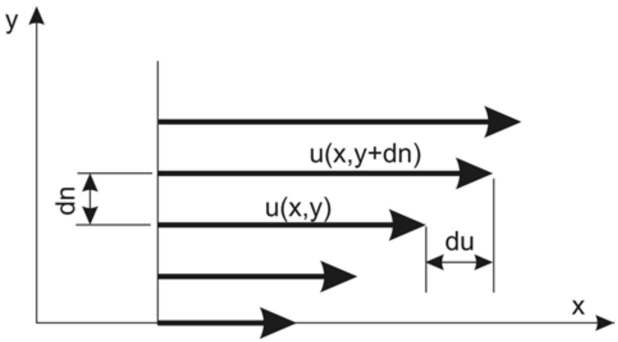
$$-\frac{\Delta V}{V} = \frac{\Delta p}{K} \Rightarrow V = V_0.(1 - \beta.\Delta p)$$

$$V = V_0.\left(1 - \frac{\Delta p}{K}\right)$$

β – koeficient stlačiteľnosti [Pa^{-1}]
K – objemový modul stlačiteľnosti [Pa]

Základné vlastnosti (pokrač.)

- Viskozita



Newtonov zákon vnút. trenia

$$\tau = -\mu \frac{\partial u}{\partial n}$$

μ – koeficient dynamickej viskozity [Pa.s]
ν – koeficient kinematickej viskozity [$m^2.s^{-1}$]

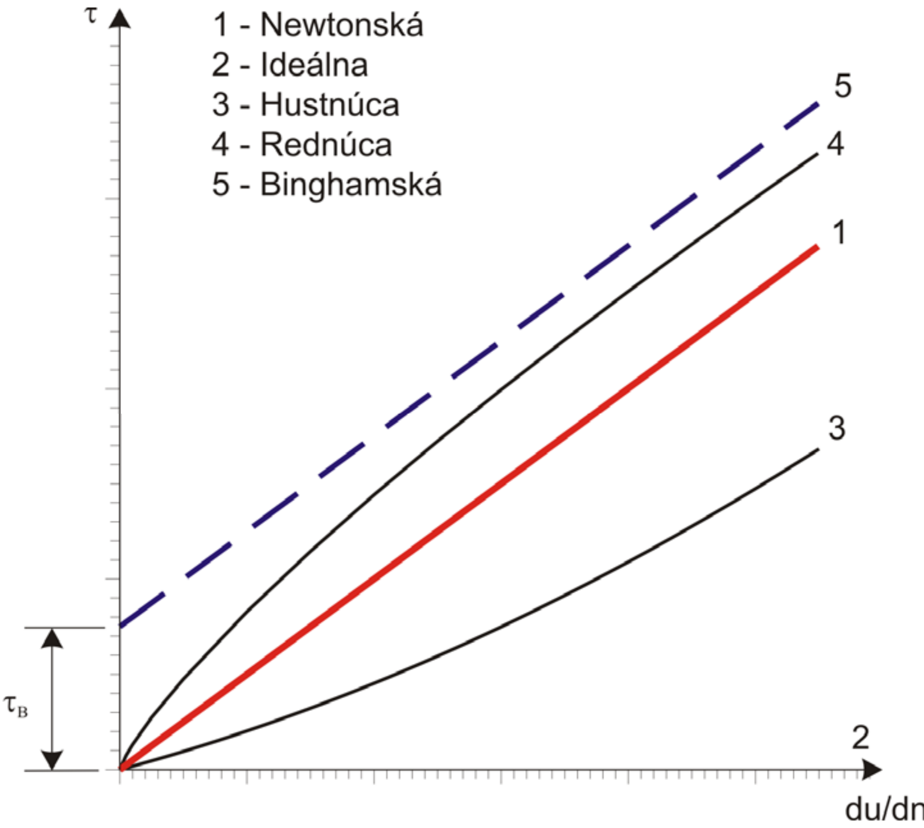
$$\nu = \frac{\mu}{\rho}$$

Druhy kvapalín

- Newtonské
- Nenewtonské
  - rednúce
  - hustnúce
  - Binghamské

$$\tau = -\mu . \left(\frac{du}{dy} \right)^n$$

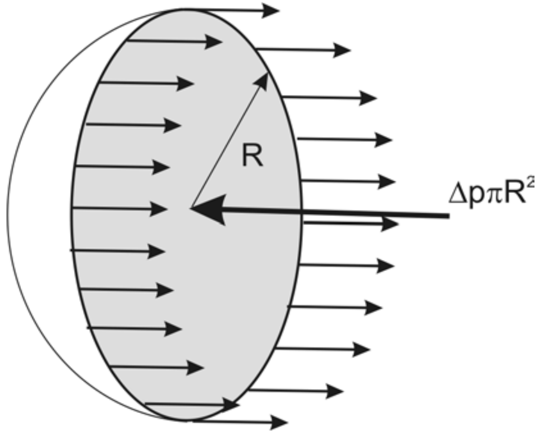
n>1 rednúce, n<1 hustnúce



Základné vlastnosti (pokrač.)

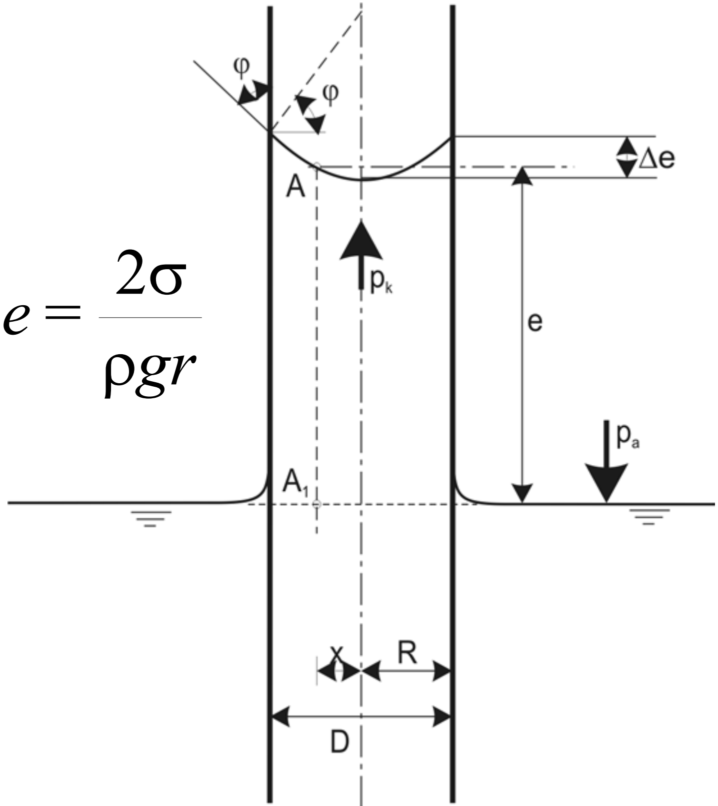
- Povrchové napätie

σ – kapilárna konštanta [N.m^{-1}]



$$2\pi R\sigma = \Delta p \pi R^2$$

$$\Delta p = \frac{2\sigma}{R}$$

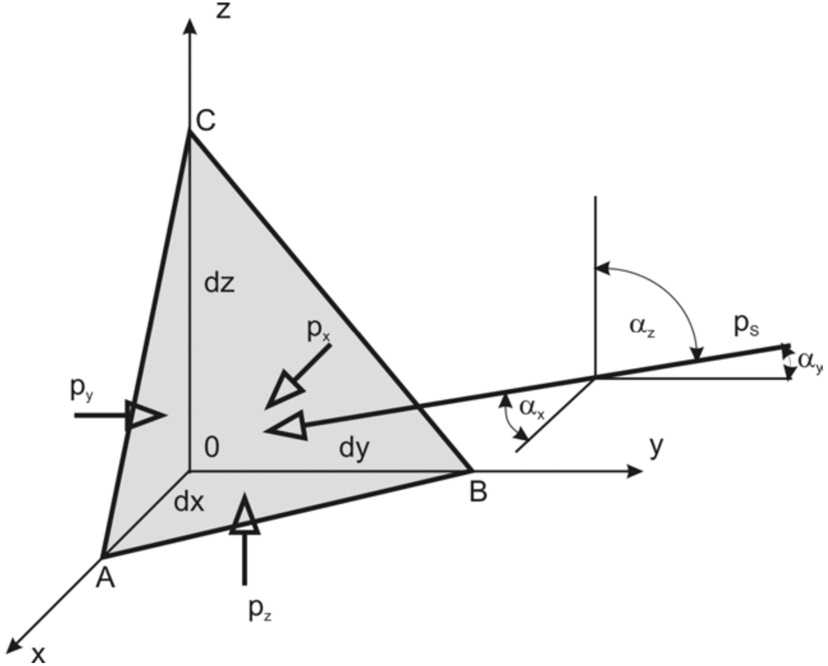


$$e = \frac{2\sigma}{\rho g r}$$

HYDROSTATIKA

$$p = \frac{dF}{dS}$$

- Tlak v bode



$$F_x = p_x \frac{dy.dz}{2} \quad F_y = p_y \frac{dx.dz}{2}$$

$$F_z = p_z \frac{dx.dy}{2} \quad F_s = pdS$$

$$p_x . \frac{dy.dz}{2} - p.dS.\cos\alpha_x = 0$$

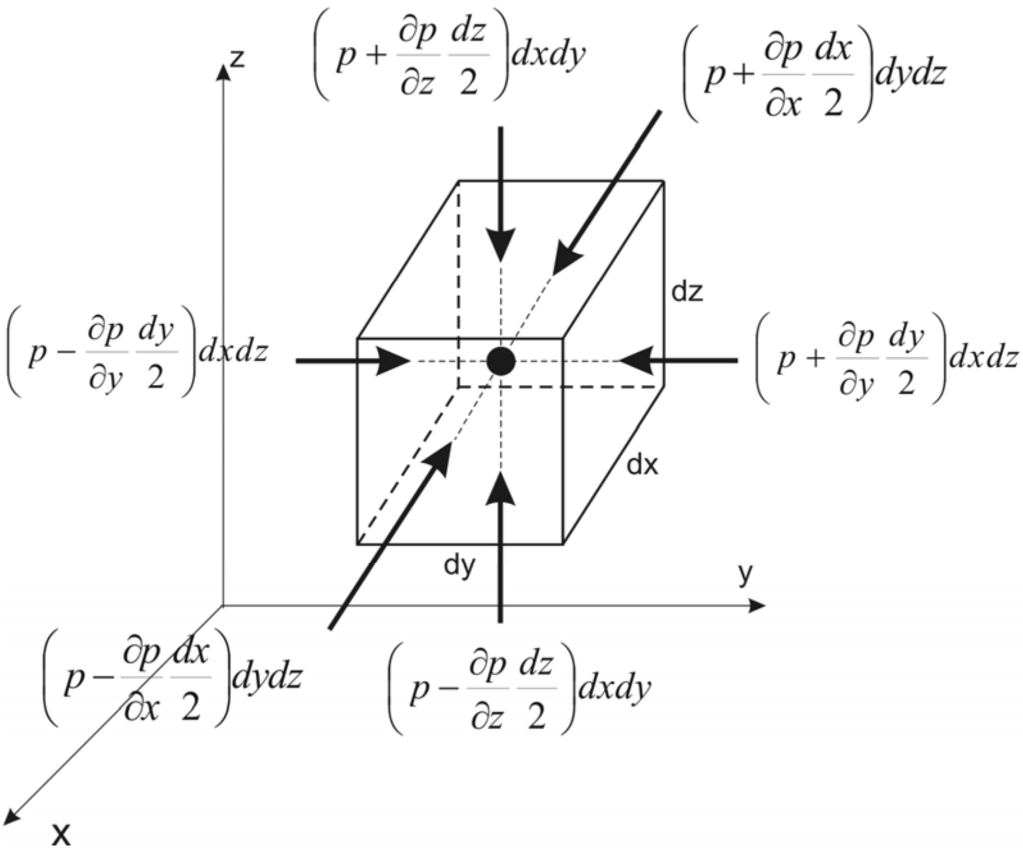
$$p_y . \frac{dx.dz}{2} - p.dS.\cos\alpha_y = 0$$

$$p_z . \frac{dx.dy}{2} - p.dS.\cos\alpha_z = 0$$

$$p_x = p_y = p_z = p$$

Rovnice rovnováhy

- Eulerove rovnice rovnováhy



$$F_{xL}=\left(p+\frac{\partial p}{\partial x}\frac{dx}{2}\right)dydz$$

$$F_{xR}=-\left(p+\frac{\partial p}{\partial x}\frac{dx}{2}\right)dydz$$

$$F_{xV}=a_x\rho\, dxdydz$$

$$-\frac{\partial p}{\partial x}+a_x\rho=0$$

$$-\frac{\partial p}{\partial y}+a_y\rho=0$$

$$-\frac{\partial p}{\partial z}+a_z\rho=0$$

Hydrostatický tlak

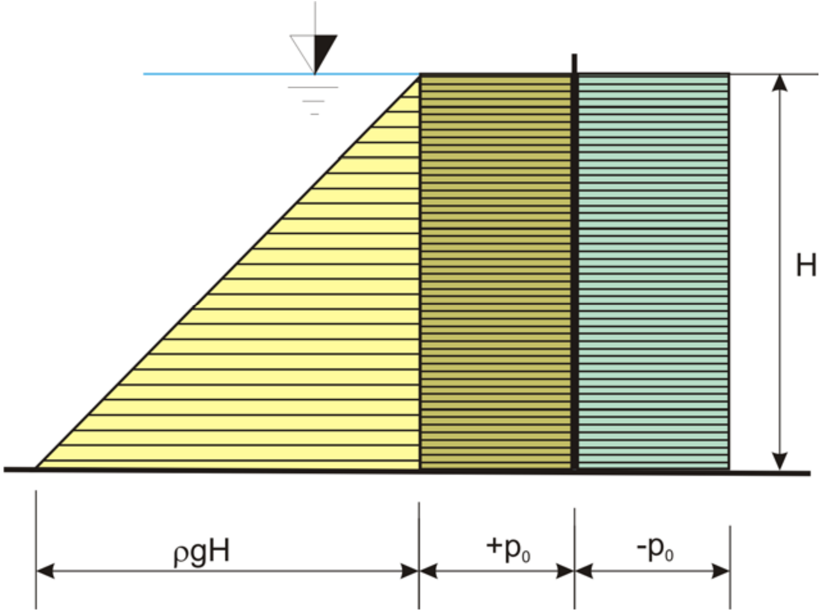
- Integráciou Eulerových rovníc

Predpoklad: $a_x = a_y = 0,\ a_z = -g$

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$$

$$-\frac{\partial p}{\partial z} = \rho g \Rightarrow dp = -\rho g dz$$

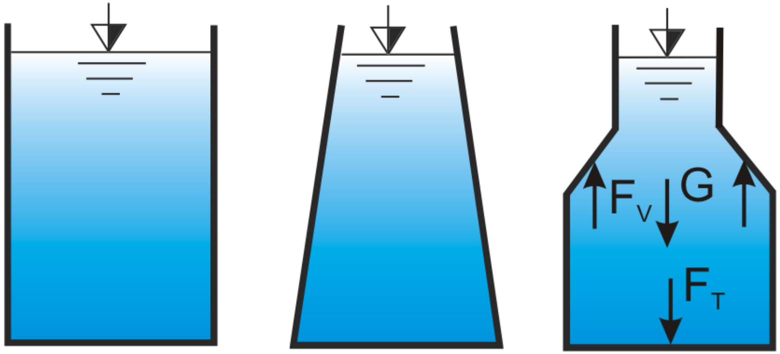
$$p = \rho . g . h + p_0$$



Tlak p_0 často zanedbávame u stavebných konštrukcií

Hydrostatický tlak

- Hydrostatické paradoxon

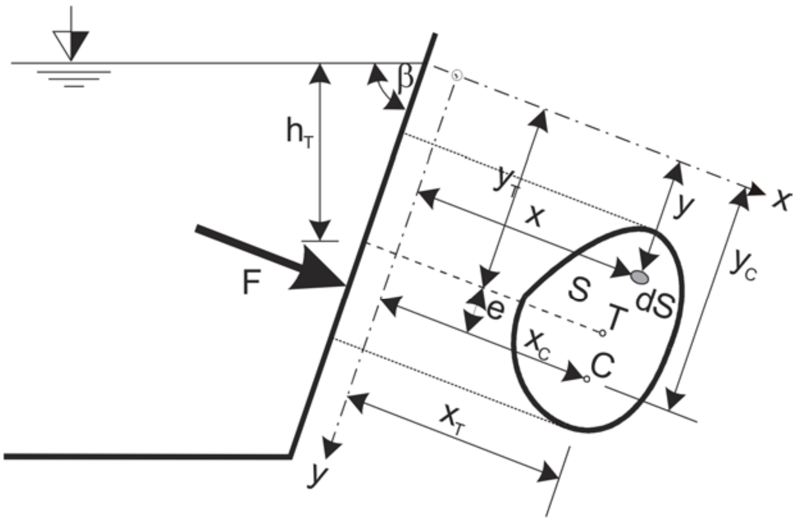


Hydrostatický tlak nie je závislý na tvare nádoby, ani na objeme kvapaliny v nej, ale iba na hĺbke a na druhu kvapaliny

$$F_T + G - F_V = 0 \Rightarrow F_T > G$$

Tlak na steny

- Určenie výslednej tlakovej sily



$$dF = p.dS = \rho g h dS$$

$$F = \rho g \sin\beta \int_S y dS$$

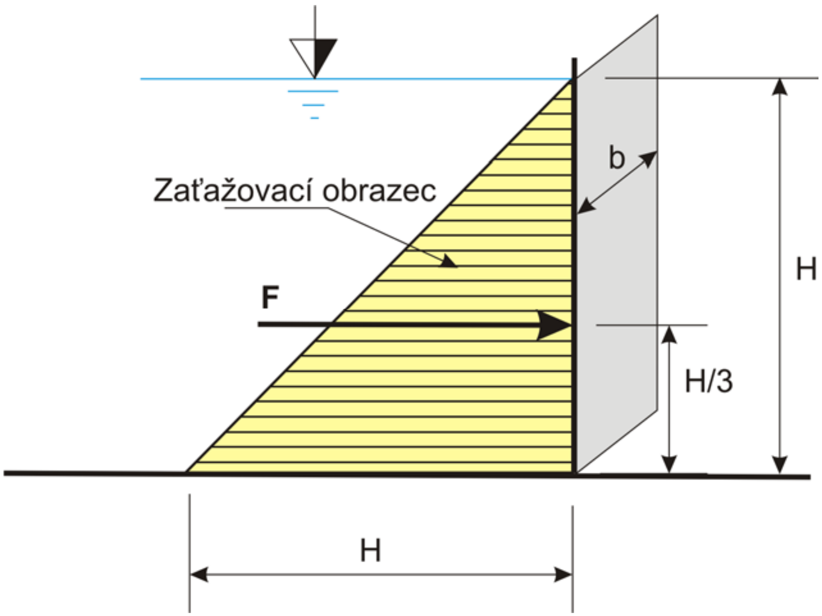
$$F = \rho g\, y_T \sin\beta S = \rho g\, h_T\, S$$

Pôsobisko sily

$$y_c = \frac{\rho g \sin\beta\, J_x}{\rho g \sin\beta S\, y_T} = \frac{J_x}{S\, y_T} = \frac{J_x}{U_x}$$

$$x_c = \frac{\rho g \sin\beta\, D_{xy}}{\rho \sin\beta S\, y_T} = \frac{D_{xy}}{S\, y_T} = \frac{D_{xy}}{U_x}$$

Tlaková sila – druhá metóda



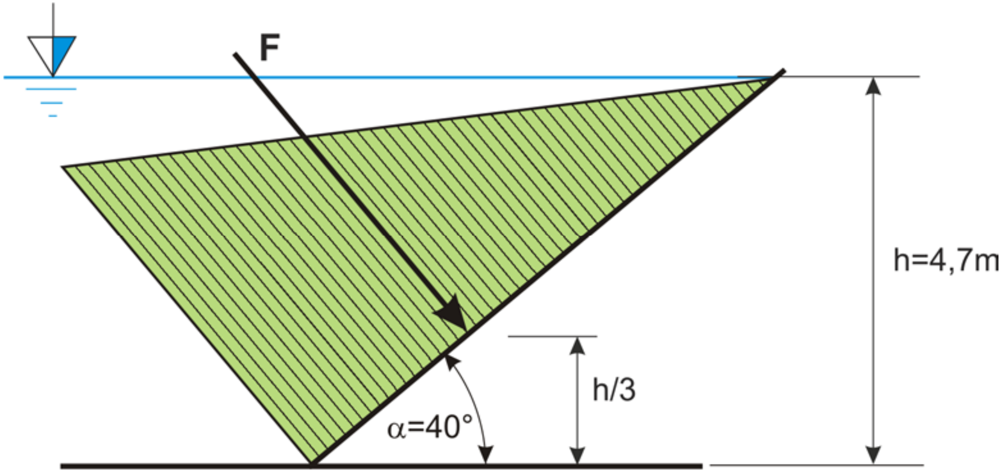
$$F = S_z . b . \rho . g$$

S_z – plocha zaťažovacieho obrazca

Pôsobisko sily je v ťažisku zaťažovacieho obrazca

Tlaková sila

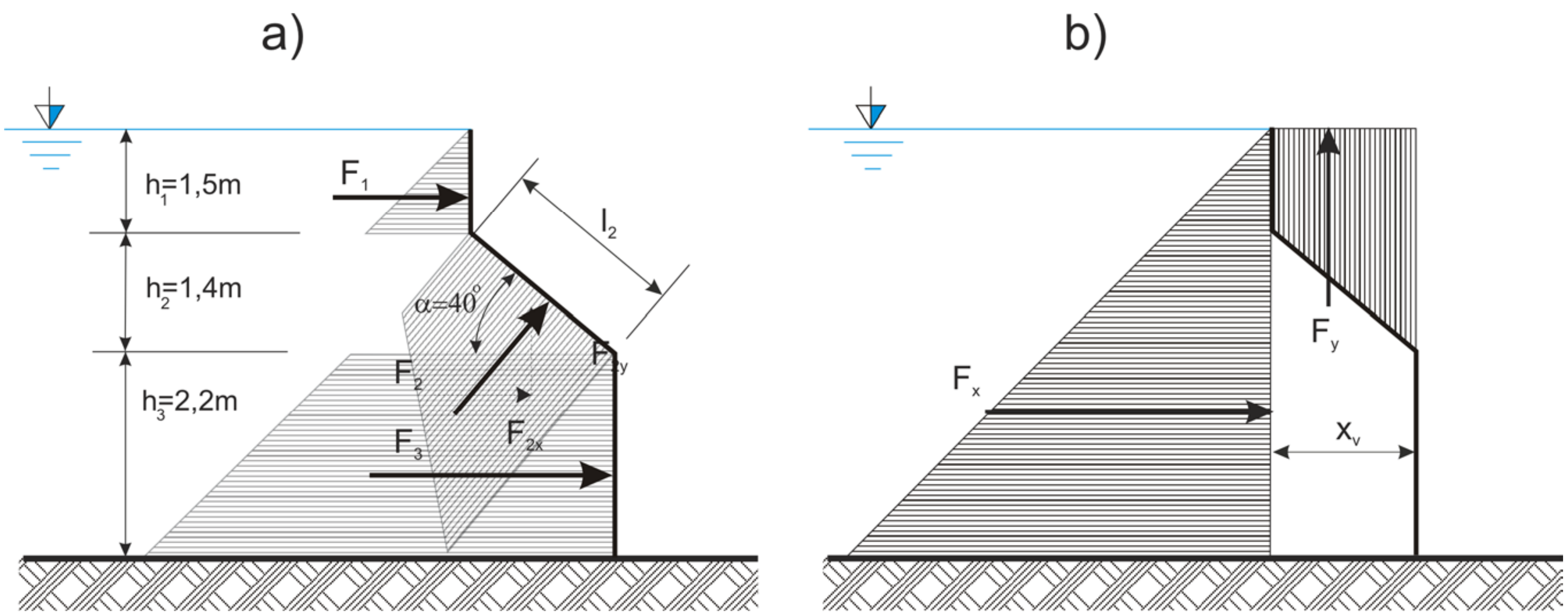
- Šikmá stena



$$l = \frac{h}{\sin\alpha} = \frac{4,7}{\sin 40°} = 7,311m \qquad S_z = \frac{1}{2}.l.h = \frac{1}{2}.7,311.4,7 = 17,182m^2$$

$$F = \rho.g.b.S_x = 1000.9,81.1.17,182 = 168,564\,kN$$

Tlaková sila – lomená stena



Príklad – plochy ZO

$$S_1 = \frac{1}{2} \cdot h_1^2 = \frac{1}{2} \cdot 1{,}5^2 = 1{,}125 m^2$$

$$S_2 = \frac{h_1 + (h_1 + h_2)}{2} \cdot l_2 = \frac{2.h_1 + h_2}{2} \cdot \frac{h_2}{\sin 40°} =$$

$$= \frac{2.1{,}5 + 1{,}4}{2} \cdot \frac{1{,}4}{\sin 40°} = 4{,}7916 m^2$$

$$S_3 = \frac{(h_1 + h_2) + (h_1 + h_2 + h_3)}{2} \cdot h_3 = \frac{2.h_1 + 2.h_2 + h_3}{2} \cdot h_3 =$$

$$= \frac{2.1{,}5 + 2.1{,}4 + 2{,}2}{2} .2{,}2 = 8{,}8 m^2$$

Príklad – pokrač.

- Sily na úseky

$$F_1 = \rho . g . b . S_1 = 1000.9{,}81.1.1{,}125 = 11{,}036 kN$$

$$F_2 = 1000.9{,}81.1.4{,}7916 = 47{,}005 kN \qquad F_3 = 86{,}328 kN$$

- Rozloženie do smerov x a y a výsledok

$$F_{2x} = F_2 . \cos 50° = 30{,}214 kN \qquad F_{2y} = F_2 . \sin 50° = 36{,}008 kN$$

$$F_x = F_1 + F_{2x} + F_3 = 11{,}036 + 30{,}214 + 86{,}328 = 127{,}578 kN \qquad F_y = F_{2y} = 36{,}008 kN$$

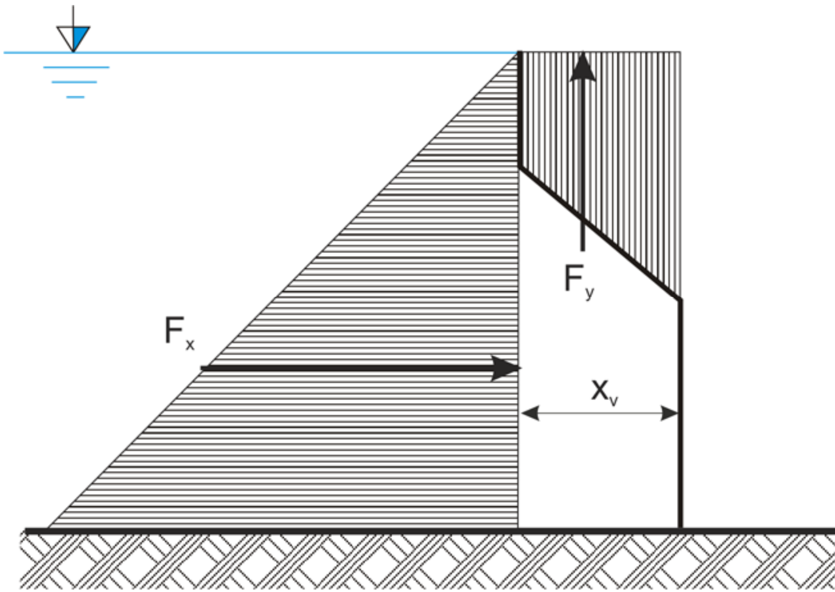
$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{127{,}578^2 + 36{,}008^2} = 132{,}563 kN$$

Príklad – druhý spôsob

- Vodorovná zložka

$$S_x = \frac{1}{2} \cdot \left(\sum_{i=1}^{3} h_i \right)^2 = \frac{1}{2} .5,1^2 = 13,005\ m^2$$

$$F_x = \rho.g.b.S_h = 1000.9,81.1.13,005 = 127,579 kN$$



- Zvislá zložka

$$V_v = \frac{h_1 + (h_1 + h_2)}{2} .x_v.b = \frac{2.h_1 + h_2}{2} \cdot \frac{h_2}{tg40°} .b =$$

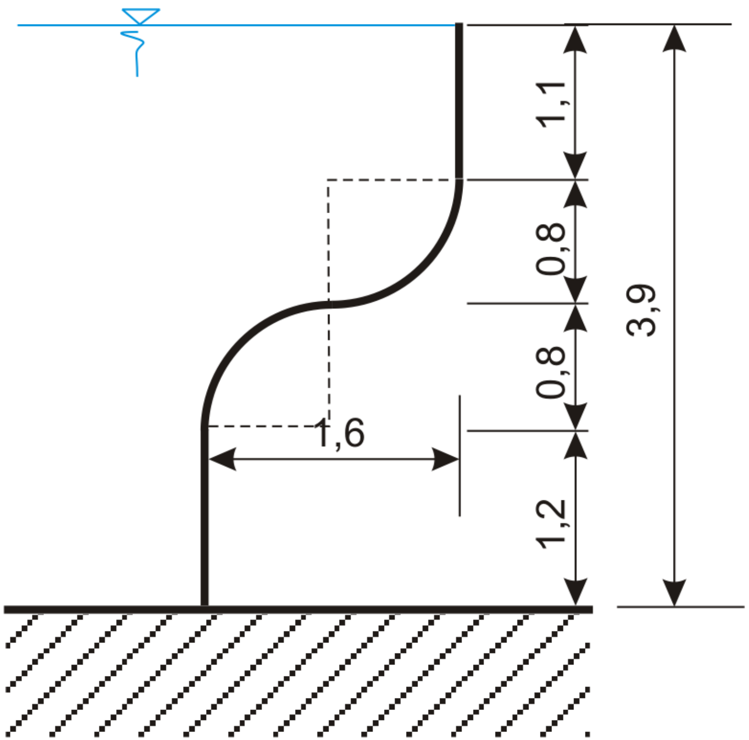
$$= \frac{2.1,5 + 1,4}{2} \cdot \frac{1,4}{tg40°} .1,0 = 3,6706\ m^3$$

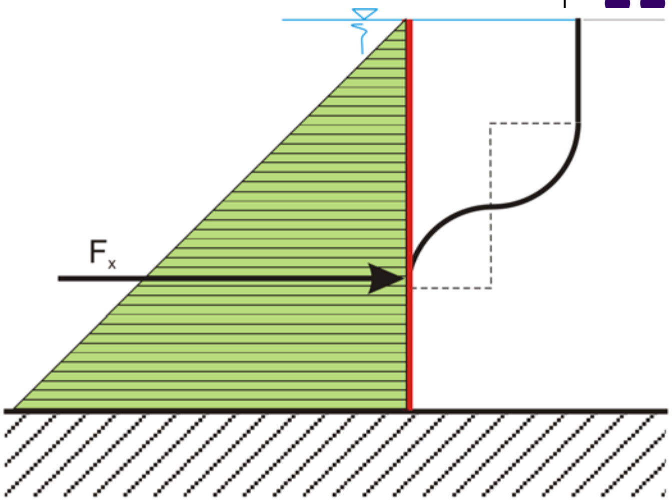
$$F_y = \rho.g.V_v = 1000.9,81.3,6706 = 36,008 kN$$

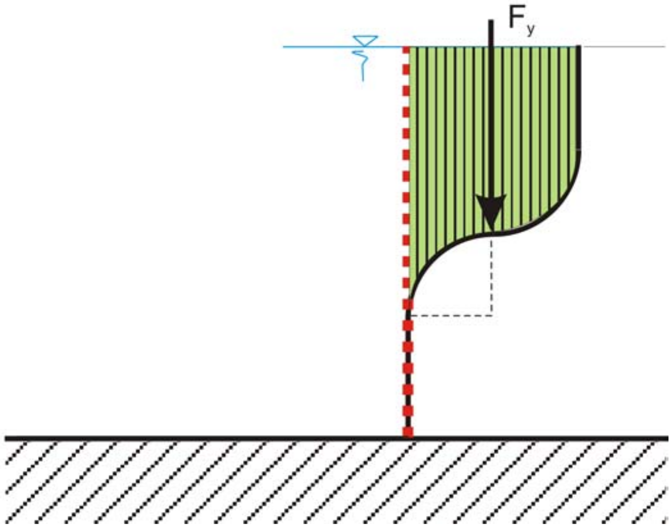
$$F = \sqrt{F_h^2 + F_v^2} = \sqrt{127,579^2 + 36,008^2} = 132,563 kN$$

Zakrivené steny

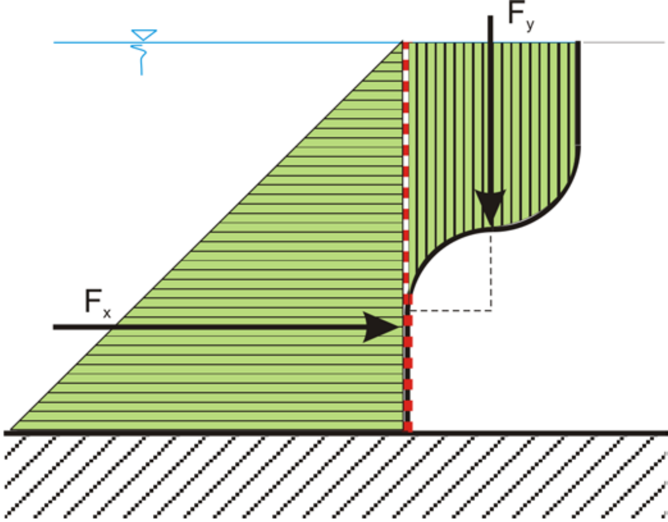
- Príklad







Riešenie príkladu



- Vodorovná zložka

$$S_x = \frac{1}{2}.3{,}9^2 = 7{,}605$$

$$F_x = \rho.g.b.S_x = 1000.9{,}81.1{,}0.7{,}605 = 74605{,}05N = 74{,}60505kN$$

- Zvislá zložka

$$S_y = 1{,}9.1.6 = 3{,}04$$

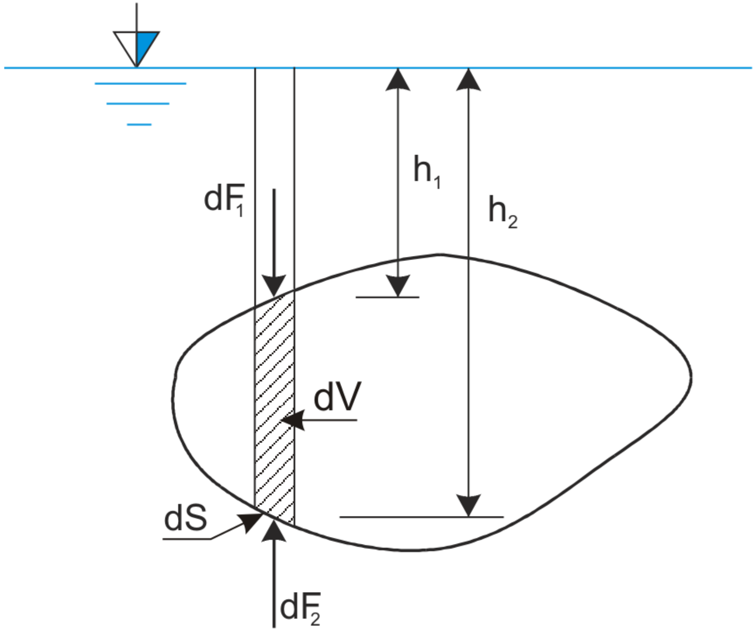
$$F_y = \rho.g.b.S_y = 1000.9{,}81.1{,}0.3{,}04 = 29822{,}4N = 29{,}8224kN$$

- Výslednica

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{74{,}60505^2 + 29{,}8224^2} = 80{,}3448kN$$

Plávanie telies

- Archimédov zákon



$$dF_1 = \left(p_0 + \rho.g.h_1\right).dS$$

$$dF_2 = \left(p_0 + \rho.g.h_2\right).dS$$

$$dF = dF_2 - dF_1 = \rho.g.\left(h_2 - h_1\right).dS$$

$$dF = \rho.g.dV$$

$$F = \int_V dF = \rho.g.\int_V dV = \rho.g.V$$

Vztlaková sila je rovná tiaži vody, vytlačenej telesom

Stabilita plávajúcich telies

- Tri druhy
  - Stabilná – metacentrum nad ťažiskom
  - Labilná – metacentrum pod ťažiskom
  - Indiferentná – metacentrum priamo v ťažisku