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Fakulta stavebná

BRIDGES

Examples

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CONTENTS

DESIGN OF STEEL AND CONCRETE COMPOSITE ROAD BRIDGE

1 PROJECT SETTINGS

The objective is to design steel and concrete composite road bridge transferring the road C9.5 and two footpaths of width 1.5 m over a river. The main steel beams, made of steel S355, with span 40 m and relative spacing 3.4 m are simply supported on concrete gravity abutments. The steel beams are connected with reinforced concrete slab of depth 250 mm, made of concrete C 30/37, by means of stud connectors ϕ 18 mm with ultimate strength f_u = 340 MPa. The bridge will be built without temporary staging. The cross section of the bridge is presented in Fig. 1.

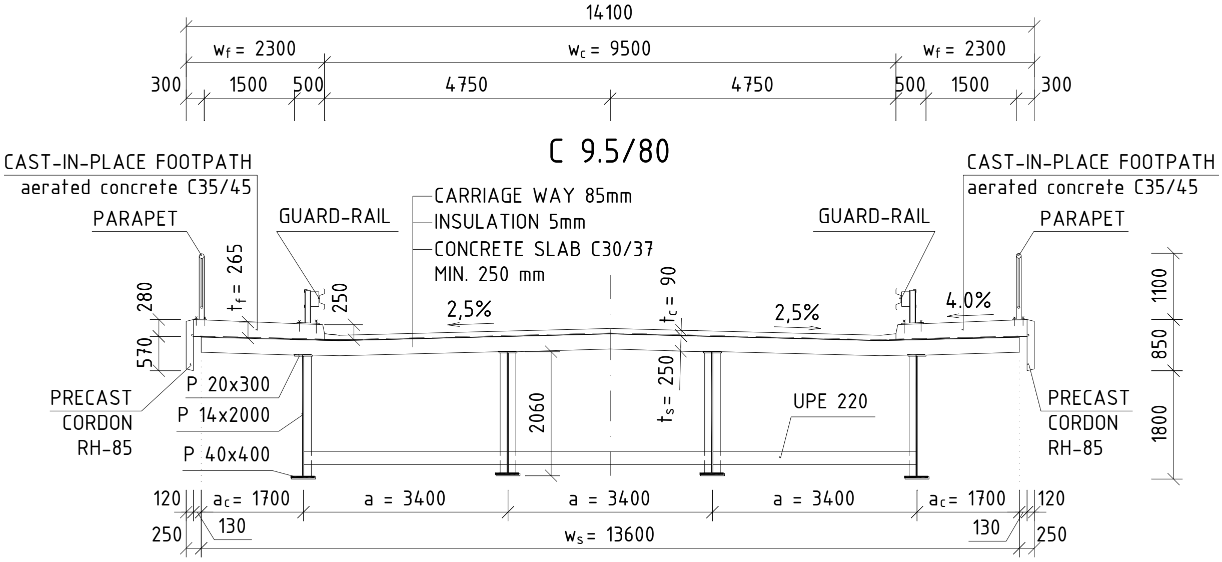


Fig. 1 Cross section of the bridge

2 LOADS AND INTERNAL FORCES

2.1 Permanent loads

The permanent loads are represented by self weight of the superstructure as well as the weight of all permanently embedded parts of bridge. With regards to un-propped construction of the bridge, the permanent loads have to be divided into two parts. The part 1 corresponds to the first construction phase, when the concrete slab is not stiffened yet and therefore all the loads have to be carried by steel beams only. The second part of permanent loads corresponds to the construction phase after the concrete slab stiffening, when the steel and concrete composite structure is acting.

According to EN 1990, Annex 2, the partial safety factor should be considered for all permanent loads by value $\gamma_{G,sup}$ = 1.35. However, the national annex to STN EN 1990, Annex 2 differs two values of partial safety factor $\gamma_{G,sup}$ – for the self weight of bridge elements made in special workshops it is defined by value 1.25, and for the self weight of bridge elements made in building site it is defined by value 1.35.

2.1.1 Part 1 (carried by steel beams)

Loads:		***Characteristic values***	γ_G	***Design values***
- concrete slab:	$t_s \cdot w_s \cdot \gamma = 0.25 \cdot 13.6 \cdot 25 =$	85.00 kN·m^{-1}	1.35	114.75 kN·m^{-1}
- steel beams:	$n \cdot A_a \cdot \gamma = 4 \cdot (0.3 \cdot 0.02 + 0.014 \cdot 1.94 + 0.4 \cdot 0.04) \cdot 80 =$	15.73 kN·m^{-1}	1.25	19.66 kN·m^{-1}
- bracing:	$g_{br} \cdot (n-1) \cdot a/(L/4) = 0.21 \cdot (4-1) \cdot 3.4/(40/4) =$	0.21 kN·m^{-1}	1.25	0.26 kN·m^{-1}
		$G_{1,k}$ = 100.94 kN·m^{-1}		$G_{1,d}$ = 134.67 kN·m^{-1}

The 1st part of permanent loads falling to one steel beam:

$$g_{1,k} = \frac{G_{1,k}}{n} = \frac{100.94}{4} = 25.24 \text{ kN·m}^{-1} \qquad g_{1,d} = \frac{G_{1,d}}{n} = \frac{134.67}{4} = 33.67 \text{ kN·m}^{-1}$$

Characteristic values of internal forces and moments in decisive cross sections:

$$M_{g1,k} = 1/8 \cdot g_{1,k} \cdot L^2 = 1/8 \cdot 25.24 \cdot 40^2 = \ 5\,048.0 \text{ kNm}$$
$$V_{g1,k} = 1/2 \cdot g_{1,k} \cdot L = 1/2 \cdot 25.24 \cdot 40 = \ 504.8 \text{ kN}$$

Design values of internal forces and moments in decisive cross sections:

$$M_{g1,Ed} = 1/8 \cdot g_{1,d} \cdot L^2 = 1/8 \cdot 33.67 \cdot 40^2 = 6\,734.0 \text{ kNm}$$
$$V_{g1,Ed} = 1/2 \cdot g_{1,d} \cdot L = 1/2 \cdot 33.67 \cdot 40 = \ 673.4 \text{ kN}$$

2.1.2 Part 2 (carried by steel and concrete composite beams)

Loads:		***Characteristic values***	γ_G	***Design values***
- carriage way:	$t_c \cdot w_c \cdot \gamma = 0.085 \cdot 9.5 \cdot 22 =$	17.77 kN·m^{-1}	1.35	23.99 kN·m^{-1}
- insulation:	$t_i \cdot w_s \cdot \gamma = 0.005 \cdot 13.6 \cdot 14 =$	0.95 kN·m^{-1}	1.25	1.19 kN·m^{-1}
- footpaths:	$2 \cdot t_f \cdot w_f \cdot \gamma = 2 \cdot 0.265 \cdot 2.3 \cdot 23 =$	28.04 kN·m^{-1}	1.35	37.85 kN·m^{-1}
- parapets:	$2 \cdot 0.3$ (estimation) =	0.60 kN·m^{-1}	1.25	0.75 kN·m^{-1}
- guard-rails:	$2 \cdot 0.4$ (estimation) =	0.80 kN·m^{-1}	1.25	1.00 kN·m^{-1}
		$G_{2,k}$ = 48.16 kN·m^{-1}		$G_{2,d}$ = 64.78 kN·m^{-1}

The 2nd part of permanent loads falling to one composite beam:

$$g_{2,k} = \frac{G_{2,k}}{n} = \frac{48.16}{4} = 12.04 \text{ kN·m}^{-1} \qquad g_{2,d} = \frac{G_{2,d}}{n} = \frac{64.78}{4} = 16.20 \text{ kN·m}^{-1}$$

Characteristic values of internal forces and moments in decisive cross sections:

$$M_{g2,k} = 1/8 \cdot g_{2,k} \cdot L^2 = 1/8 \cdot 12.04 \cdot 40^2 = \ 2\,408.0 \text{ kNm}$$
$$V_{g2,k} = 1/2 \cdot g_{2,k} \cdot L = 1/2 \cdot 12.04 \cdot 40 = \ 240.8 \text{ kN}$$

Design values of internal forces and moments in decisive cross sections:

$$M_{g2,Ed} = 1/8 \cdot g_{2,d} \cdot L^2 = 1/8 \cdot 16.20 \cdot 40^2 = 3\,240.0 \text{ kNm}$$
$$V_{g2,Ed} = 1/2 \cdot g_{2,d} \cdot L = 1/2 \cdot 16.20 \cdot 40 = \ 324.0 \text{ kN}$$

2.2 Variable traffic loads

Variable loads are represented mainly by vertical effects of road traffic load, which should be generally considered by means of four loading models, defined in STN EN 1991-2. The main loading system is represented by Load model 1 (LM1), consisting of concentrated tandem system (TS) and uniformly distributed load (UDL system) situated in carriage way divided into several notional lanes. The LM1 expresses majority of passenger and freight service and should be used for verification of global and local effects of traffic as well.

The load model 2 (LM2) is characterised by single axle vehicle and it may be determining mainly for short structural members with length 3 – 7 m. It is intended mainly for verification of local effects of traffic.

The load model 3 (LM3) is intended for modelling special vehicles (e.g. for industrial traffic), moving pass the special permitted road tracks reserved for heavy loads. This model should be used for verification of global and local effects of traffic as well.

The load model 4 (LM4) expresses the load caused by moving mass of people and it should by used for general verification of a structure.

It may be simply proved that the LM2 and LM4 have less unfavourable effects on the superstructure strain than the LM1. The LM3 will not be considered for simplification. Consequently, only effects of the LM1 will be taken into account. These effects should be combined with horizontal effects of traffic load (braking forces) and pedestrian load of footpaths as well. While the braking forces will be neglected, the effect of pedestrian load of footpaths will be taken into account by reduced combination value.

Transversal arrangement of LM1 should be considered using the influence line of transversal distribution obtained by method of rigid transversal bracing (Fig. 2).

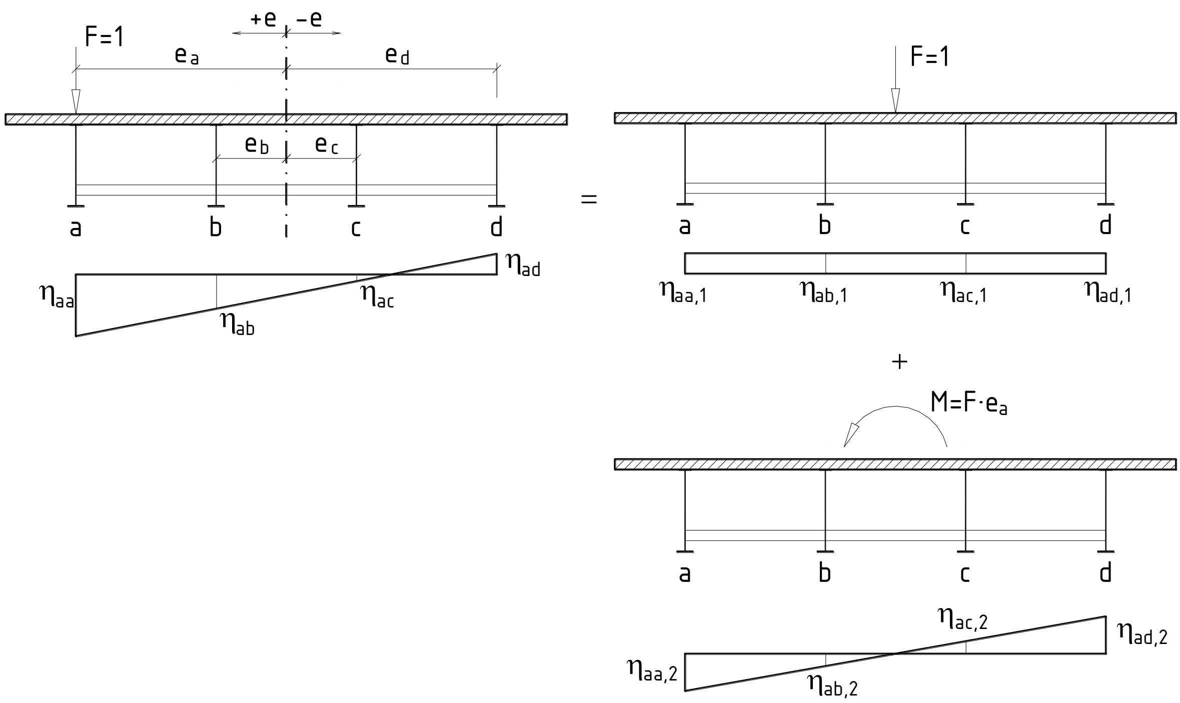


Fig. 2 Influence line of transversal distribution for the outer beam “a”

Taking into account the bridge symmetry, only beams "a" and "b" need to be verified. Coordinates of the influence line for the outer beam "a" are given by equation:

$$\eta_{ai} = \eta_{ai,1} + \eta_{ai,2} = \frac{I_{ya}}{\sum_{i=a}^{d} I_{yi}} + \frac{e_a \cdot I_{ya}}{\sum_{i=a}^{d} I_{yi} \cdot e_i^2} \cdot e_i = \frac{1}{n} + \frac{e_a}{\sum_{i=a}^{d} e_i^2} \cdot e_i$$

$$\left.\begin{aligned}
\eta_{aa} &= \frac{1}{4} + \frac{1.5a}{\left[(1.5a)^2 + (0.5a)^2\right] \cdot 2} \cdot 1.5a = 0.7 \\
\eta_{ab} &= \frac{1}{4} + \frac{1.5a}{\left[(1.5a)^2 + (0.5a)^2\right] \cdot 2} \cdot 0.5a = 0.4 \\
\eta_{ac} &= \frac{1}{4} + \frac{1.5a}{\left[(1.5a)^2 + (0.5a)^2\right] \cdot 2} \cdot (-0.5a) = 0.1 \\
\eta_{ac} &= \frac{1}{4} + \frac{1.5a}{\left[(1.5a)^2 + (0.5a)^2\right] \cdot 2} \cdot (-1.5a) = -0.2
\end{aligned}\right\} \sum_{i=a}^{d} \eta_{ai} = 1.0$$

Similarly, coordinates of the influence line for the inner beam "b" are given by equation:

$$\eta_{bi} = \eta_{bi,1} + \eta_{bi,2} = \frac{I_{yb}}{\sum_{i=a}^{d} I_{yi}} + \frac{e_b \cdot I_{yb}}{\sum_{i=a}^{d} I_{yi} \cdot e_i^2} \cdot e_i = \frac{1}{n} + \frac{e_b}{\sum_{i=a}^{d} e_i^2} \cdot e_i$$

$$\left.\begin{aligned}
\eta_{ba} &= \frac{1}{4} + \frac{0.5a}{\left[(1.5a)^2 + (0.5a)^2\right] \cdot 2} \cdot 1.5a = 0.4 \\
\eta_{bb} &= \frac{1}{4} + \frac{0.5a}{\left[(1.5a)^2 + (0.5a)^2\right] \cdot 2} \cdot 0.5a = 0.3 \\
\eta_{bc} &= \frac{1}{4} + \frac{0.5a}{\left[(1.5a)^2 + (0.5a)^2\right] \cdot 2} \cdot (-0.5a) = 0.2 \\
\eta_{bd} &= \frac{1}{4} + \frac{0.5a}{\left[(1.5a)^2 + (0.5a)^2\right] \cdot 2} \cdot (-1.5a) = 0.1
\end{aligned}\right\} \sum_{i=a}^{d} \eta_{ai} = 1.0$$

Arrangement of the Load Model 1 together with the pedestrian load of footpaths in transversal direction is presented in Fig. 3. All the loads are considered in the most unfavourable positions with regard to the particular influence line of transversal distribution. The lightening effects are not taken into account, however, the tandem systems are always considered as complete. Classification factors α_{Qi} and α_{qi} may be generally considered within an interval 0.8 – 1.0. The values applied here are considered according to national annex to STN EN 1991-2.

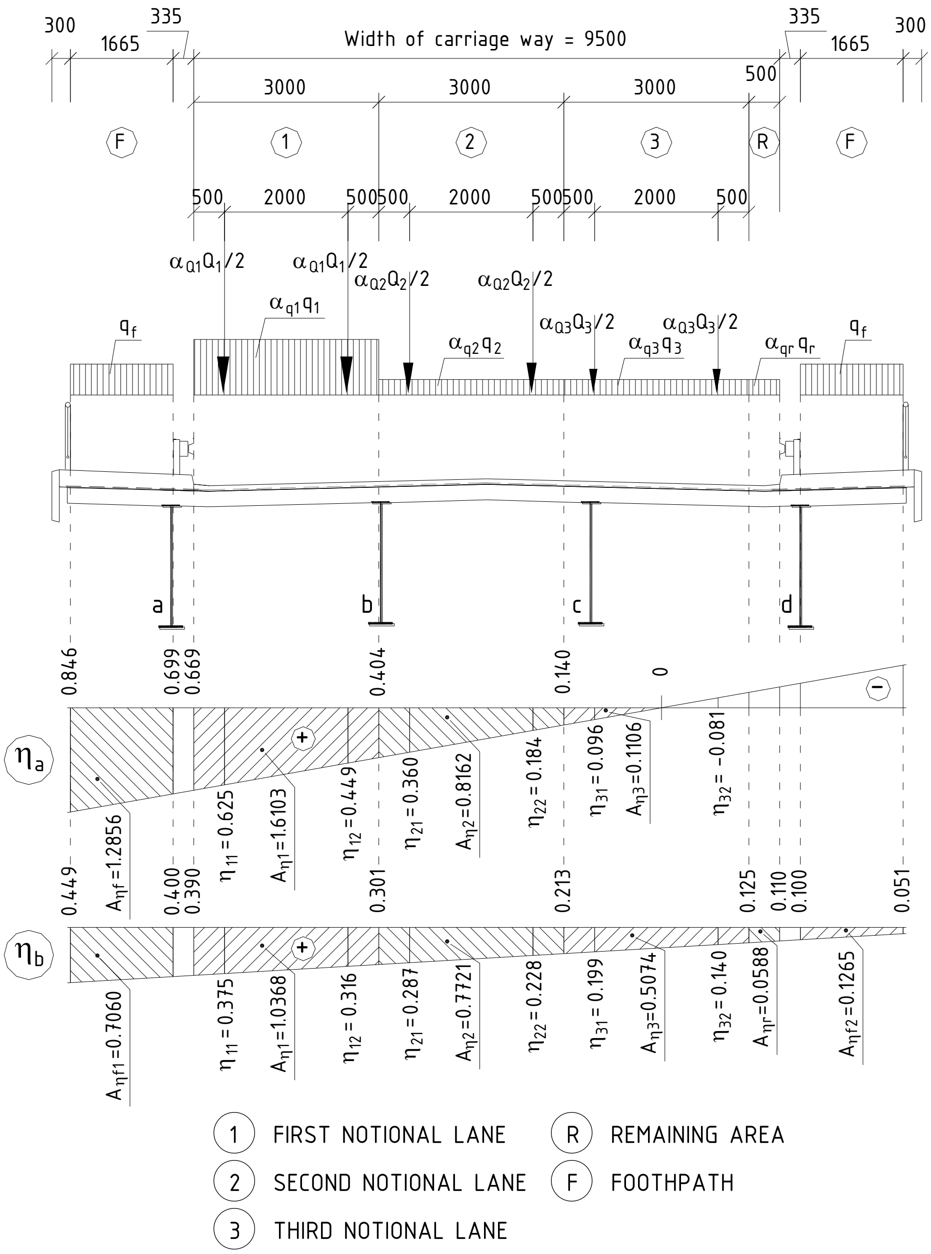


Fig. 3 Arrangement of LM1 with pedestrian load and coordinates of the influence lines

The outer beam "a" is affected by following loads:

- from tandem system (TS):

$$F_{TS} = \alpha_{Q1} Q_1/2 \cdot (\eta_{11} + \eta_{12}) + \alpha_{Q2} Q_2/2 \cdot (\eta_{21} + \eta_{22}) + \alpha_{Q3} Q_3/2 \cdot (\eta_{31} + \eta_{32}) =$$
$$= 0.9 \cdot 300/2 \cdot (0.625 + 0.449) + 0.9 \cdot 200/2 \cdot (0.360 + 0.184) + 0.9 \cdot 100/2 \cdot (0.096 - 0.081) =$$
$$= 194.63\,\text{kN}$$

- from UDL system:

$$p_{UDL} = \alpha_{q1} q_1 \cdot A_{\eta 1} + \alpha_{q2} q_2 \cdot A_{\eta 2} + \alpha_{q3} q_3 \cdot A_{\eta 3} =$$
$$= 0.9 \cdot 9.0 \cdot 1.6103 + 1.0 \cdot 2.5 \cdot 0.8162 + 1.0 \cdot 2.5 \cdot 0.1106 = 15.36\ \text{kN}{\cdot}\text{m}^{-1}$$

- from pedestrian load of footpaths:

$$p_f = q_f \cdot A_{\eta f} = 3.0 \cdot 1.2856 = 3.86\ \text{kN}{\cdot}\text{m}^{-1}$$

The inner beam "b" is affected by following loads:

- from tandem system (TS):

$$F_{TS} = \alpha_{Q1}Q_1/2\cdot(\eta_{11}+\eta_{12})+\alpha_{Q2}\,Q_2/2\cdot(\eta_{21}+\eta_{22})+\alpha_{Q3}Q_3/2\cdot(\eta_{31}+\eta_{32})=$$
$$=0.9\cdot 300/2\cdot(0.375+0.316)+0.9\cdot 200/2\cdot(0.287+0.228)+0.9\cdot 100/2\cdot(0.199+0.140)=$$
$$=154.89\,\text{kN}$$

- from UDL system:

$$p_{UDL} = \alpha_{q1}q_1\cdot A_{\eta 1}+\alpha_{q2}q_2\cdot A_{\eta 2}+\alpha_{q3}q_3\cdot A_{\eta 3}+\alpha_{qr}q_r\cdot A_{\eta r} =$$
$$=0.9\cdot 9.0\cdot 1.0368+1.0\cdot 2.5\cdot 0.7721+1.0\cdot 2.5\cdot 0.5074+1.0\cdot 2.5\cdot 0.0588=11.74\,\text{kN}\cdot\text{m}^{-1}$$

- from pedestrian load of footpaths:

$$p_f = q_f\cdot(A_{\eta f1}+A_{\eta f2})=3.0\cdot(0.7060+0.1265)=2.50\ \text{kN}\cdot\text{m}^{-1}$$

Characteristic values of internal forces and moments in decisive cross sections:

The influence lines for calculation of internal forces and moments in longitudinal direction are presented in Fig. 4. All the loads from transversal distribution are considered in the most unfavourable positions with regard to the particular influence line. With regards to the higher values of all particular loads, only the outer beam "a" needs to be further evaluated.

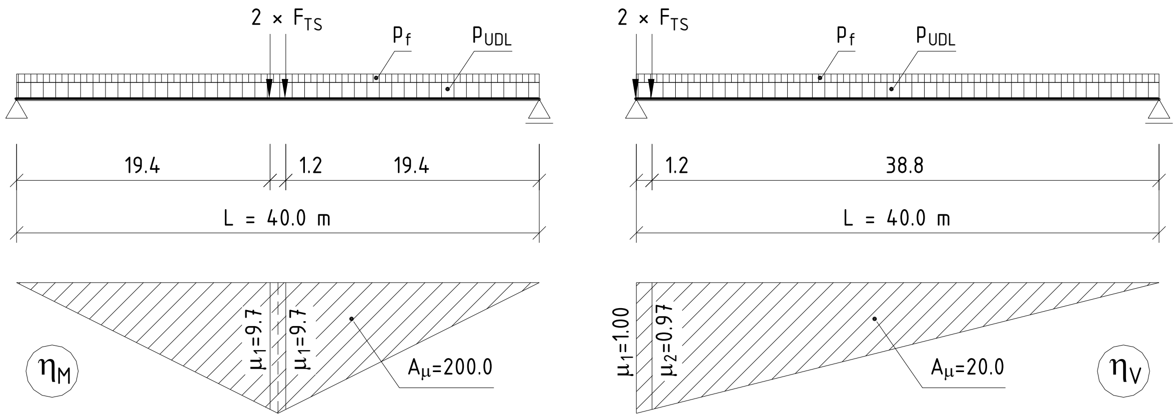


Fig. 4 Influence lines for bending moment and shear force

$$M_{LM1,k} = M_{TS,k}+M_{UDL,k} = 2\cdot F_{TS}\cdot\mu_1 + p_{UDL}\cdot A_\mu = 2\cdot 194.63\cdot 9.7+15.36\cdot 200.0=$$
$$=3\,775.82 + 3\,072.00 = 6\,847.82\ \text{kNm}$$
$$M_{f,k} = p_f\cdot A_\mu = 3.86\cdot 200.0 = 772.00\ \text{kNm}$$
$$V_{LM1,k} = V_{TS,k}+V_{UDL,k} = F_{TS}\cdot(\mu_1+\mu_2)+p_{UDL}\cdot A_\mu = 194.63\cdot(1.0+0.97)+15.36\cdot 20.0=$$
$$=383.42 + 307.20 = 690.62\ \text{kN}$$
$$V_{f,k} = p_f\cdot A_\mu = 3.86\cdot 20.0 = 77.20\ \text{kN}$$

According to STN EN 1990, Annex 2, the load model LM1 consisting of TS system and UDL system and the reduced pedestrian load of footpaths make together the load group "gr1a", which should be considered as one multi-part variable load. The resultant characteristic values of internal forces and moments corresponding to the load group "gr1a" are:

$$M_{gr1a,k} = M_{TS,k}+M_{UDL,k}+M_{f,k} = 3\,775.82 + 3\,072.00 + 772.00 = 7619.82\ \text{kNm}$$
$$V_{gr1a,k} = V_{LM1,k}+V_{f,k} = 690.62 + 77.20 = 767.82\ \text{kN}$$

3 CROSS-SECTION DESIGN FOR ULTIMATE LIMIT STATE

According to STN EN 1994-2, the effects of creep and shrinkage of concrete, as well as the effects of sequence of construction and temperature effects may be neglected in analysis for verifications of ultimate limit states other than fatigue, providing that all cross-sections of the composite member are in class 1 or 2. Then, the verification consists in comparison of the design bending moment and the design shear force, respectively, caused by the most unfavourable combination of actions, with the design bending resistance of composite cross-section and shear resistance of the steel beam web, respectively.

3.1 Combination of loads for the ultimate limit states

Design value of bending moment affecting the outer beam "a" at the mid-span is:

$$M_{Ed} = \sum\left(\gamma_{Gj} \cdot M_{gk,j}\right) + \gamma_{Q,gr1a} \cdot M_{gr1a,k} = M_{g1,Ed} + M_{g2,Ed} + \gamma_{Q,gr1a} \cdot M_{gr1a,k} =$$
$$= 6\,734.0 + 3\,240.0 + 1.35 \cdot 7\,619.82 = 20\,260.76 \text{ kNm}$$

Design value of shear force affecting the outer beam "a" at the support is:

$$V_{Ed} = \sum\left(\gamma_{Gj} \cdot V_{gk,j}\right) + \gamma_{Q,gr1a} \cdot V_{gr1a,k} = V_{g1,Ed} + V_{g2,Ed} + \gamma_{Q,gr1a} \cdot V_{gr1a,k} =$$
$$= 673.4 + 324.0 + 1.35 \cdot 767.82 = 2\,033.96 \text{ kN}$$

3.2 Characteristics of cross-section

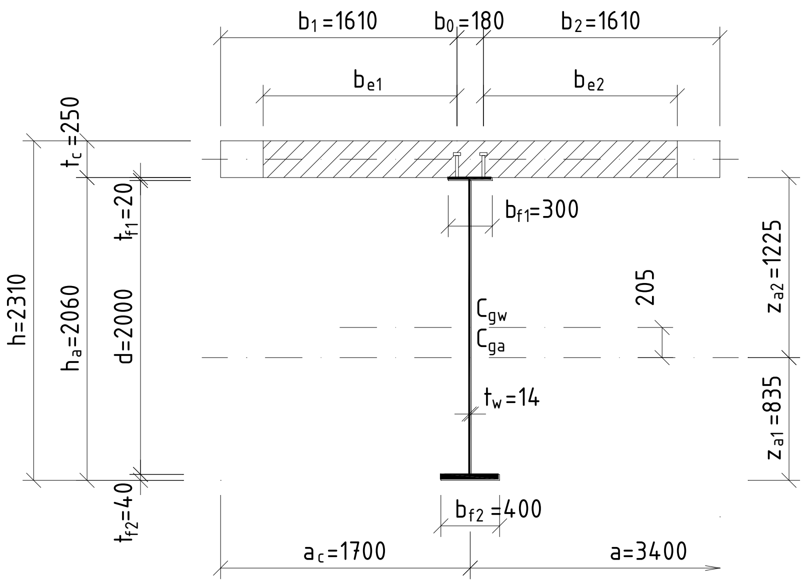
Effective width of the concrete slab at the mid-span of beam "a" is defined by an expression

$b_{eff} = b_0 + \sum b_{ei}$.

Estimating distance of the outer queues of studs b_0 = 180 mm, the partial effective widths are:

$b_{e,1} = b_{e,2} = L_e / 8 = 40\,000 / 8 = 5\,000$ mm > $b_1 = b_2 = 1\,610$ mm $\Rightarrow b_{e,1} = b_{e,2} = 1\,610$ mm .

Consequently, the total effective width of concrete slab is b_{eff} = 3400 mm, regardless of the distance of outer queues of studs b_0.



Steel beam:

A_a = 0.050 m^2

I_{ya} = 0.029997 m^4

Concrete slab:

A_c = 0.850 m^2

I_{yc} = 0.004427 m^4

Fig. 5 Cross section of the composite beam

Material parameters of steel: (according to STN EN 1993-2)

Class of steel: S355
Characteristic yield strength: f_{yk} = 355 MPa
Partial safety factor: γ_{M0} =1.0, γ_{M1} =1.1
Design yield strength: $f_{yd} = f_{yk} / \gamma_M$ = 355 MPa
Modulus of elasticity: E = 210 000 MPa

Material parameters of concrete: (according to STN EN 1992-1-1)

Class of concrete: C30/37
Characteristic cylinder compression strength: f_{ck} = 30 MPa
Partial safety factor: γ_C = 1.5
Design cylinder compression strength: $f_{cd} = f_{ck} / \gamma_C$ = 20 MPa
Modulus of elasticity: E_{cm} = 33 000 MPa

3.3 Assessment of the cross-section bending resistance in the mid-span

Location of plastic neutral axis and classification of composite cross section

Basically, there are three possible locations of the neutral axis. Either it lies in concrete slab, or in upper flange of steel beam, or it passes through the web of the steel beam. Its location results from the equilibrium condition of normal forces in the cross section at ultimate limit state.

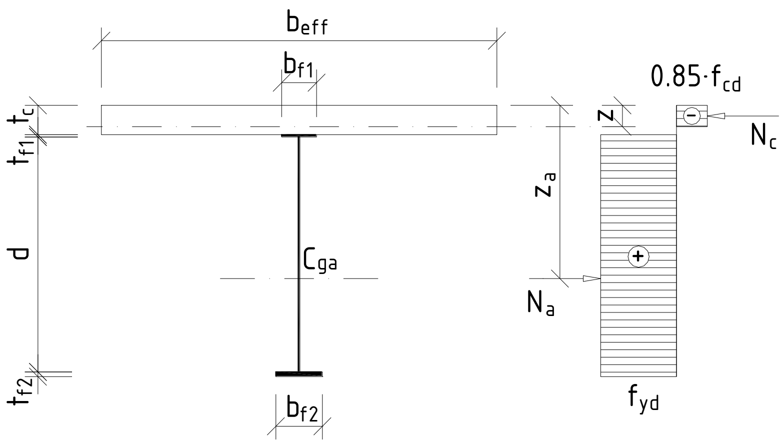


Fig. 6 Tenseness at ultimate limit state for neutral axis lying in concrete slab

If $A_c \cdot 0.85\, f_{cd} \geq A_a \cdot f_{yd}$, the neutral axis lies in concrete slab (Fig. 6) and its location results from the following equilibrium condition

$$N_c = N_a$$

$$b_{eff} \cdot z \cdot 0.85 f_{cd} = A_a \cdot f_{yd} \quad \Rightarrow \quad z = \frac{A_a \cdot f_{yd}}{b_{eff} \cdot 0.85 f_{cd}}$$

Te whole steel part is in tension, and so the classification of cross section is not needed. The plastic bending resistance results from the moment equilibrium condition in the cross section

$$M_{pl,Rd} = N_a \cdot \left(z_a - 0.5z\right) = A_a \cdot f_{yd} \cdot \left(z_a - 0.5z\right).$$

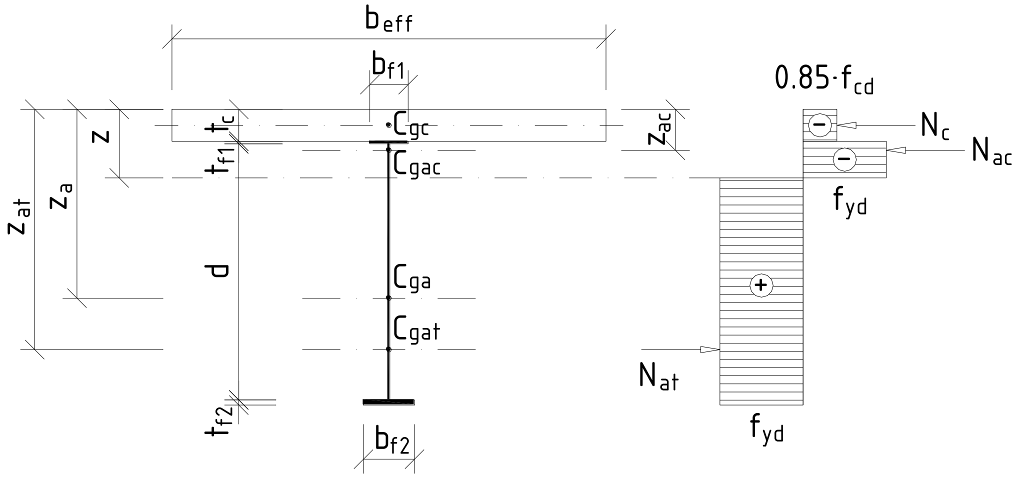


Fig. 7 Tenseness at ultimate limit state for neutral axis lying in steel beam

If $A_c \cdot 0.85\ f_{cd} < A_a \cdot f_{yd}$, the neutral axis is situated in the steel beam (Fig. 7) and the equilibrium condition of normal forces at ultimate limit state is defined by equation

$$N_c + N_{ac} = N_a$$

$$A_c \cdot 0.85 f_{cd} + A_{ac} \cdot f_{yd} = A_{at} \cdot f_{yd} = \left(A_a - A_{ac}\right) \cdot f_{yd}$$

$$A_c \cdot 0.85 f_{cd} + 2 \cdot A_{ac} \cdot f_{yd} = A_a \cdot f_{yd} \quad \Rightarrow \qquad A_{ac} = \frac{A_a \cdot f_{yd} - A_c \cdot 0.85 f_{cd}}{2 \cdot f_{yd}}$$

When the area of steel compression part $A_{ac} \leq b_{f1} \cdot t_{f1}$, the neutral axis lies in the upper flange, in other case it passes through the web of steel beam. The location of neutral axis is then given as follows

$$z = \begin{cases} t_c + \dfrac{A_{ac}}{b_{f1}} & \text{for } A_{ac} \leq b_{f1} \cdot t_{f1} \\[2ex] t_c + t_{f1} + \dfrac{A_{ac} - b_{f1} \cdot t_{f1}}{t_w} & \text{for } A_{ac} > b_{f1} \cdot t_{f1} \end{cases}$$

In the case of neutral axis laying in the upper flange of steel beam the classification of cross section is not needed. If the neutral axis passes through the web, classification of the compressed part of the web according to STN EN 1993-1-1 is necessary. The steel compression flange that is restrained from buckling by effective attachment to the concrete flange by shear connectors may be assumed to be in class 1, if the spacing of connectors is in accordance with the shear connection detailing given in 6.6.5.5 STN EN 1994-2.

Providing that the compression part of steel web is classified into class 1 or 2, the plastic bending resistance results from the moment equilibrium condition in the composite cross section

$$M_{pl,Rd} = N_{at} \cdot \left(z_{at} - 0.5 t_c\right) - N_{ac} \cdot \left(z_{ac} - 0.5 t_c\right) = N_a \cdot \left(z_a - 0.5 t_c\right) - 2 N_{ac} \cdot \left(z_{ac} - 0.5 t_c\right)$$

$$M_{pl,Rd} = \left[A_a \cdot \left(z_a - 0.5 t_c\right) - 2 \cdot A_{ac} \cdot \left(z_{ac} - 0.5 t_c\right)\right] \cdot f_{yd}$$

Applying the approach mentioned above, for the cross section presented in Fig. 5 it is valid

$$A_c \cdot 0.85 f_{cd} = 0.85 \cdot 0.85 \cdot 20 = 14.45 \text{ MN} < A_a \cdot f_{yd} = 0.05 \cdot 355 = 17.75 \text{ MN}$$

$\Rightarrow$ neutral axis lies in steel beam

The area of steel compression part is

$$A_{ac} = \frac{A_a \cdot f_{yd} - A_c \cdot 0.85 f_{cd}}{2 \cdot f_{yd}} = \frac{17.75 - 14.45}{2 \cdot 355} = 0.00465 \text{ m}^2$$

$$A_{ac} = 0.00465 \text{ m}^2 < b_{f1} \cdot t_{f1} = 0.3 \cdot 0.02 = 0.006 \text{ m}^2$$ $\Rightarrow$ neutral axis lies in upper flange

The location of plastic neutral axis is

$$z = t_c + \frac{A_{ac}}{b_{f1}} = 0.25 + \frac{0.00465}{0.3} = 0.2655 \text{ m}$$

$$z_{ac} = \frac{z + t_c}{2} = \frac{0.2655 + 0.25}{2} = 0.25775 \text{ m}$$

Since the neutral axis lies in the upper flange of steel beam, therefore the cross section may be assumed to be in class 1.

Check of the plastic bending resistance of composite cross section in the mid-span:

$$M_{pl,Rd} = \left[A_a \cdot (z_a - 0.5 t_c) - 2 \cdot A_{ac} \cdot (z_{ac} - 0.5 t_c)\right] \cdot f_{yd}$$

$$M_{pl,Rd} = \left[0.05 \cdot (1.475 - 0.5 \cdot 0.25) - 2 \cdot 0.00465 \cdot (0.25775 - 0.5 \cdot 0.25)\right] \cdot 355 \cdot 10^3$$

$$M_{pl,Rd} = \underline{\mathbf{23\ 524.23\ kNm}} > M_{Ed} = \underline{\mathbf{20\ 260.76\ kNm}}$$

3.4 Assessment of shear resistance of the cross-section at the support

The shear resistance of the cross section is given by shear resistance of the web:

$$V_{pl,Rd} = A_w \cdot f_{yd} / \sqrt{3} = 2.0 \cdot 0.014 \cdot 355 \cdot 10^3 / \sqrt{3} = \underline{\mathbf{5\ 738.86\ kN}} > V_{Ed} = \underline{\mathbf{2\ 033.96\ kN}}$$

The web of steel beam is satisfactory. However, the effect of shear buckling should be checked yet according to STN EN 1993-1-5. The shear buckling coefficient k_τ is (for the web without longitudinal stiffeners and with rigid transverse stiffeners at distances a = 10 m):

$$k_\tau = 5.34 + 4 \cdot (h_w / a)^2 = 5.34 + 4 \cdot (2.0/10)^2 = 5.5$$

The web slenderness parameter may be taken as:

$$\bar{\lambda}_w = \frac{h_w / t_w}{37.4 \cdot \varepsilon \cdot \sqrt{k_\tau}} = \frac{2000/14}{37.4 \cdot \sqrt{\frac{235}{355}} \cdot \sqrt{5.5}} = 2.002 > 1{,}08 \Rightarrow$$

Contribution from the web to shear buckling resistance is

$$\chi_w = \frac{1.37}{0.7 + \bar{\lambda}_w} = \frac{1.37}{0.7 + 2.002} = 0.507$$

Neglecting the contribution of flanges χ_f, the reduction factor for shear buckling is

$$\chi_V = \chi_w + \chi_f = 0.507 + 0.0 = 0.507$$

The design shear resistance (with shear buckling effect) is

$$V_{b,Rd} = \frac{\chi_V \cdot f_{yw} \cdot h_w \cdot t_w}{\sqrt{3} \cdot \gamma_{M1}} = \frac{0.507 \cdot 355 \cdot 10^3 \cdot 2.0 \cdot 0.014}{\sqrt{3} \cdot 1.1} = 2645.09 \text{ kN}$$

$V_{b,Rd}$ = **2 645.09 kN** > V_{Ed} = **2 033.96 kN**

4 CHECK OF CROSS-SECTION FOR SERVICEABILITY LIMIT STATE

In the case of composite members with cross-section classified in class 1 or 2, it should be further proved that the nominal stresses in the limiting cross sections resulting from the characteristic load combinations (partial safety factors $\gamma_{Gi} = \gamma_{Qi} = 1.0$) do not exceed the design strength of materials, i.e. the beam stays in elastic conditions. The calculation of stresses should take into account (apart from other effects) the effect of shear lag, creep and shrinkage of concrete and sequence of construction.

4.1 Characteristics of composite cross section for short-term load effects

Effective width of the concrete slab at the mid-span of beam "a" is the same as in ultimate limit states. Calculation of the elastic resistance to bending is based on the effective cross-section, in which the concrete part is transformed to equivalent steel part by means of the modular ratio for short-term loading $n_0 = E_a / E_{cm}$.

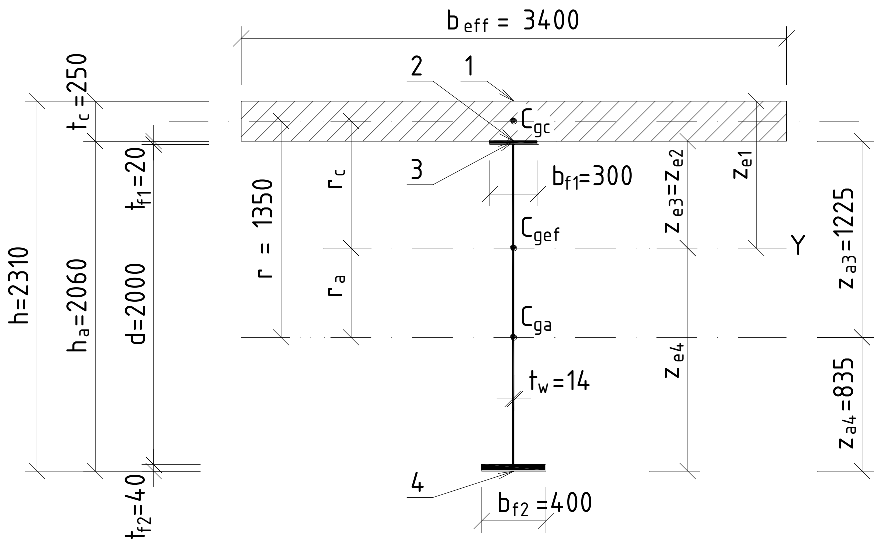


Fig. 8 Effective cross section of the composite beam

1(2) – upper (bottom) edge of concrete slab, 3(4) – upper (bottom) edge of steel beam
C_{ga} (C_{gc}) [C_{gef}] – centre of gravity of steel beam (concrete slab) [effective cross-section]

Characteristics of concrete slab:

$A_c = 3.4 \cdot 0.25 = 0.850\ \text{m}^2$

$I_{yc} = \frac{1}{12} \cdot 3.4 \cdot 0.25^3 = 0.004427\ \text{m}^4$

Characteristics of steel beam:

$A_a = 0.4 \cdot 0.04 + 0.014 \cdot 2.0 + 0.3 \cdot 0.02 = 0.050\ \text{m}^2$

$z_{a4} = \frac{0.4 \cdot 0.04 \cdot 0.02 + 0.014 \cdot 2.0 \cdot 1.04 + 0.3 \cdot 0.02 \cdot 2.05}{0.05} = 0.835\ \text{m}$

$z_{a3} = 2.060 - 0.835 = 1.225\ \text{m}$

$$I_{ya} = \frac{1}{12} \cdot \left(0.4 \cdot 0.04^3 + 0.014 \cdot 2.0^3 + 0.3 \cdot 0.02^3\right) + 0.4 \cdot 0.04 \cdot 0.815^2 + 0.014 \cdot 2.0 \cdot 0.205^2 +$$
$$+ 0.3 \cdot 0.02 \cdot 1.215^2 = 0.029997\ \text{m}^4$$

Characteristics of effective cross section of composite beam:

$n_0 = E_a / E_{cm} = 210 / 33 = 6.364$
$A_{ef} = A_a + A_c / n_0 = 0.05 + 0.85 / 6.364 = 0.18357\ \text{m}^2$

$r_c = \frac{A_a \cdot r}{A_{ef}} = \frac{0.05 \cdot 1.35}{0.18357} = 0.368\ \text{m}$

$r_a = \frac{A_c / n_0 \cdot r}{A_{ef}} = \frac{0.85/6.36 \cdot 1.35}{0.18357} = 0.982\ \text{m}$

$I_{y,ef} = I_{ya} + I_{yc} / n_0 + A_a \cdot r_a^2 + A_c / n_0 \cdot r_c^2 = I_{ya} + I_{yc} / n_0 + A_{ef} \cdot r_a \cdot r_c$
$I_{y,ef} = 0.029997 + 0.004427/6.364 + 0.18357 \cdot 0.982 \cdot 0.368 = 0.097018\ \text{m}^4$

4.2 Characteristics of composite cross section for long-term load effects

Concrete is a material underlying to creep, which may be simply defined as an increase of deformation in time under persistent long-term loading. This effect may be mathematically described as a degradation of modulus of elasticity, which in composite girder results in gradual migration of normal stresses from the concrete slab to the steel beam. The effects of creep may be taken into account by using modular ratios n_L for calculation of the effective cross-section characteristics. The modular ratios depending on the type of loading are given by expression

$$n_L = n_0 \left(1 + \psi_L \cdot \phi_t\right)$$

where n_0 is the modular ratio for short-term loading, ϕ_t is the creep coefficient $\phi(t,t_0)$ depending on the age (t) of concrete at the moment considered and the age (t_0) at loading, and ψ_L is the creep multiplier depending on the type of loading, which may be taken as 1.1 for permanent loads, 0.55 for primary and secondary effects of shrinkage and 1.5 for pre-stressing by imposed deformations.

According to EN 1992-1-1, Annex B, the creep coefficient $\phi(t,t_0)$ may be calculated from expression:

$$\phi(t,t_0) = \phi_0 \cdot \beta_c(t - t_0),$$

where ϕ_0 is the theoretical creep coefficient given by expression

$$\phi_0 = \phi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0).$$

Coefficient ϕ_{RH} expresses an effect of relative humidity of environment RH (%) on the theoretical creep coefficient and is given by expression

$$\phi_{RH} = \begin{cases} 1 + \dfrac{1 - RH/100}{0,1 \cdot \sqrt[3]{h_0}} & for\ f_{cm} \le 35\ MPa \\ \left[1 + \dfrac{1 - RH/100}{0,1 \cdot \sqrt[3]{h_0}} \cdot \alpha_1 \right] \cdot \alpha_2 & for\ f_{cm} \ge 35\ MPa \end{cases}$$

where h_0 (mm) is equivalent depth of slab $h_0 = \dfrac{2A_c}{u}$, A_c is area of the concrete slab and u is the perimeter of member that is in contact with atmosphere.

Coefficient $\beta(f_{cm})$ expresses an effect of concrete strength on the theoretical creep coefficient and is given by expression

$$\beta(f_{cm}) = \frac{16,8}{\sqrt{f_{cm}}}$$

where $f_{cm} = f_{ck} + 8$ is the mean compression strength of concrete at the age 28 days.

Coefficient $\beta(t_0)$ expresses an effect of the age of concrete at loading on the theoretical creep coefficient and is given by expression

$$\beta(t_0) = \frac{1}{0,1 + t_0^{0,20}} .$$

Coefficient $\beta_c(t\text{-}t_0)$ describes development of creep in time and may by estimated by using following expression

$$\beta_c(t - t_0) = \left[\frac{t - t_0}{\beta_H + t - t_0} \right]^{0,3}$$

Coefficient β_H depends on relative humidity RH (%) and equivalent size of member h_0 and is given by expression

$$\beta_H = \begin{cases} 1,5 \cdot \left[1 + (0,012 \cdot RH)^{18} \right] \cdot h_0 + 250 \le 1500 & for\ f_{cm} \le 35 \\ 1,5 \cdot \left[1 + (0,012 \cdot RH)^{18} \right] \cdot h_0 + 250 \cdot \alpha_3 \le 1500 & for\ f_{cm} \ge 35 \end{cases}$$

Coefficients α_1, α_2, α_3 taking into account the concrete strength are defined as follows

$$\alpha_1 = \left(\frac{35}{f_{cm}} \right)^{0.7} \qquad \alpha_2 = \left(\frac{35}{f_{cm}} \right)^{0.2} \qquad \alpha_3 = \left(\frac{35}{f_{cm}} \right)^{0.5} .$$

Creep coefficient for second part of permanent loads:

On the assumption that the second part of permanent loads will start to affect the composite girder two months after concreting (t_0 = 60 days), the bridge will be introduced into service four months after concreting (t_{in} = 120 days) and the planned service life of the bridge is 100 years (t_{fin} = 36 525 days), then at the relative humidity RH = 80% it will be valid:

$$h_0 = \frac{2A_c}{u} = \frac{2 \cdot 0.85}{2 \cdot 3.4} = 0.25 \text{ m}$$

f_{cm} = 30 + 8 = 38 MPa > 35 MPa

$$\alpha_1 = \left(\frac{35}{38}\right)^{0.7} = 0.94406 \qquad \alpha_2 = \left(\frac{35}{38}\right)^{0.2} = 0.98369 \qquad \alpha_3 = \left(\frac{35}{38}\right)^{0.5} = 0.95971$$

$$\beta_H = 1{,}5 \cdot \left[1 + (0{,}012 \cdot 80)^{18}\right] \cdot 250 + 250 \cdot 0.95971 = 794.779 < 1500$$

$$\beta_c(t - t_0) = \left[\frac{120 - 60}{794.779 + 120 - 60}\right]^{0,3} = 0.45070 \qquad \text{for } t = 120 \text{ days}$$

$$\beta_c(t - t_0) = \left[\frac{36\,525 - 60}{794.779 + 36\,525 - 60}\right]^{0,3} = 0.99355 \qquad \text{for } t = 36\,525 \text{ days}$$

$$\beta(t_0) = \frac{1}{0{,}1 + 60^{0{,}20}} = 0.42231$$

$$\beta(f_{cm}) = \frac{16{,}8}{\sqrt{38}} = 2.72532$$

$$\phi_{RH} = \left[1 + \frac{1 - 85/100}{0{,}1 \cdot \sqrt[3]{250}} \cdot 0.94406\right] \cdot 0.98369 = 1.20481$$

$$\phi_0 = 1.20481 \cdot 2.72532 \cdot 0.42231 = 1.38665$$

The creep coefficient at the time of introducing into service (t = 120 days) is:

$$\phi_t = \phi(120, 60) = 1.38665 \cdot 0.45070 = 0.62496$$

The creep coefficient at the end of service life (t = 36 525 days) is:

$$\phi_t = \phi(36\,525, 60) = 1.38665 \cdot 0.99355 = 1.37771$$

Creep coefficient for effect of shrinkage of concrete:

Shrinkage of concrete is a long-term process, independent on acting load, which is characteristic by gradual decreasing volume of concrete. Since the steel beam restrains to free deformation of the concrete slab, there occur additional strains in the composite girder. According to STN EN 1994-2, the creep coefficient due to shrinkage should be calculated on the assumption that the shrinkage will start from the first day after concreting (t_0 = 1 day).

$$\beta_c(t - t_0) = \left[\frac{120 - 1}{794.779 + 120 - 1}\right]^{0,3} = 0.54251 \qquad \text{for } t = 120 \text{ days}$$

$$\beta_c(t - t_0) = \left[\frac{36\,525 - 1}{794.779 + 36\,525 - 1}\right]^{0,3} = 0.99356 \qquad \text{for } t = 36\,525 \text{ days}$$

$$\beta(t_0) = \frac{1}{0{,}1 + 1^{0{,}20}} = 0.90909$$

$$\phi_0 = 1.20481 \cdot 2.72532 \cdot 0.90909 = 2.98499$$

The creep coefficient at the time of introducing into service (t = 120 days) is:

$$\phi_t = \phi(120, 60) = 2.98499 \cdot 0.54251 = 1.61939$$

The creep coefficient at the end of service life (t = 36 525 days):

$$\phi_t = \phi(36\,525, 60) = 2.98499 \cdot 0.99356 = 2.96577$$

Modular ratios for long-term load effects:

- for the second part of permanent loads at the time t = 120 days:
  $$n_L = n_0\left(1 + \psi_L \cdot \phi_t\right) = 6.364 \cdot \left(1 + 1.1 \cdot 0.62496\right) = 10.738$$
- for the second part of permanent loads at the time t = 36 525 days:
  $$n_L = n_0\left(1 + \psi_L \cdot \phi_t\right) = 6.364 \cdot \left(1 + 1.1 \cdot 1.37771\right) = 16.008$$
- for the shrinkage effects at the time t = 120 days:
  $$n_L = n_0\left(1 + \psi_L \cdot \phi_t\right) = 6.364 \cdot \left(1 + 0.55 \cdot 1.61939\right) = 12.032$$
- for the shrinkage effects at the time t = 36525 days:
  $$n_L = n_0\left(1 + \psi_L \cdot \phi_t\right) = 6.364 \cdot \left(1 + 0.55 \cdot 2.96577\right) = 16.744$$

Characteristics of effective cross-section of composite beam for long-term load effects:

The effective cross-section characteristics corresponding to calculated modular ratios n_L are summarised in Tab. 1.

Tab. 1 Effective cross-section characteristics

Load		2nd part of permanent loads			Shrinkage		
Age of bridge	[days]	60	120	36525	1	120	36525
$\phi(t, t_0)$	[-]	-	0.625	1.378	-	1.619	2.966
ψ_L	[-]	-	1.100	1.100	-	0.550	0.550
$n_0 = E_a/E_{cm}$	[-]	6.364	-	-	6.364	-	-
$n_L = n_0 \cdot (1+\psi_L \cdot \phi(t, t_0))$	[-]	-	10.738	16.008	-	12.032	16.744
A_a	[$\cdot 10^{-3}$m^2]	50.000	50.000	50.000	50.000	50.000	50.000
A_c/n	[$\cdot 10^{-3}$m^2]	133.571	79.156	53.100	133.571	70.648	50.765
$A_{ef} = A_a + A_c/n$	[$\cdot 10^{-3}$m^2]	183.571	129.156	103.100	183.571	120.648	100.765
$r_c = A_a \cdot a/A_{ef}$	[m]	0.368	0.523	0.655	0.368	0.560	0.670
$r_a = A_c \cdot a/(n \cdot A_{ef})$	[m]	0.982	0.827	0.695	0.982	0.791	0.680
I_a	[$\cdot 10^{-3}$m^4]	29.997	29.997	29.997	29.997	29.997	29.997
I_c/n	[$\cdot 10^{-3}$m^4]	0.696	0.412	0.277	0.696	0.368	0.264
$A_{ef} \cdot a_c \cdot a_a$	[$\cdot 10^{-3}$m^4]	66.325	55.864	46.946	66.325	53.376	45.922
I_{ef}	[$\cdot 10^{-3}$m^4]	97.018	86.274	77.220	97.018	83.741	76.184
z_{e1}	[m]	0.493	0.648	0.780	0.493	0.685	0.795
z_{e2}	[m]	0.243	0.398	0.530	0.243	0.435	0.545
z_{e3}	[m]	0.243	0.398	0.530	0.243	0.435	0.545
z_{e4}	[m]	1.817	1.662	1.530	1.817	1.625	1.515

4.3 Classification of effective cross-section

It is sufficient to verify classification of the effective cross-section only for the case with maximum distance of the upper edge of steel beam from the elastic neutral axis z_{e3}. The limiting value of the width to thickness ratio for class 3 is defined STN EN 1993-1-1 in dependence on the stress ratio

$$\textit{when } \psi > -1: \quad c/t \le \frac{42 \cdot \varepsilon}{0,67 + 0,33 \cdot \psi}$$

$$\textit{when } \psi \le -1: \quad c/t \le 62 \cdot \varepsilon \cdot (1-\psi) \cdot \sqrt{(-\psi)}$$

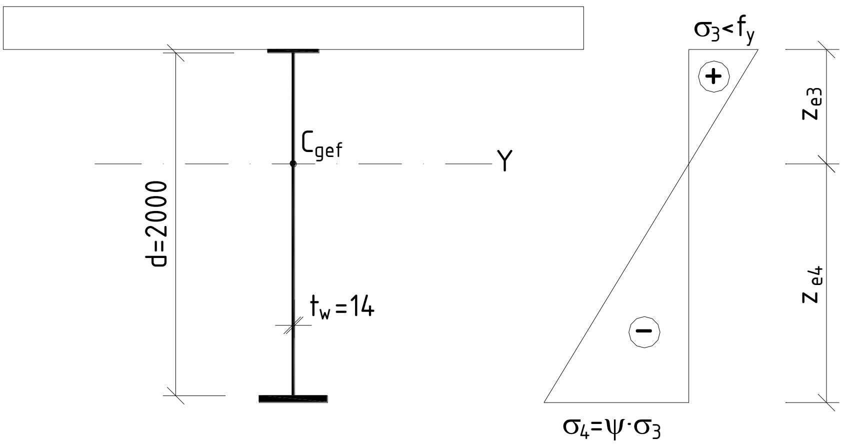


Fig. 9 Stress distribution in the cross-section

From the classification viewpoint, the most unfavourable location of elastic neutral axis is defined for effects of shrinkage at the end of service life (t = 100 years) by the value z_{e3} = 0.545 m. It means that the maximum stresses will be at the bottom edge of steel beam (Fig. 9) and the stress ratio ψ may be expressed

$$\psi = \frac{-\sigma_4}{\sigma_3} = \frac{-z_{e4}}{z_{e3}} = \frac{-1515}{545} = -2.78,$$

and the limiting proportion is

$$62 \cdot \varepsilon \cdot (1-\psi) \cdot \sqrt{-\psi} = 62 \cdot \sqrt{235/355} \cdot (1 + 2.78) \cdot \sqrt{2.78} = 317.93$$

Providing that the effective depth of fillet weld connecting the flanges and web of steel beam is a = 7 mm, the slenderness ratio of the web is

$$\beta_w = \frac{c}{t_w} = \frac{2000 - 2 \cdot 7 \cdot \sqrt{2}}{14} = 141.44 < 317.93 \quad \Rightarrow$$ the cross-section may be classified as class 3

4.4 Calculation of normal stresses

4.4.1 Stresses caused by part 1 of permanent loads (carried by steel beam)

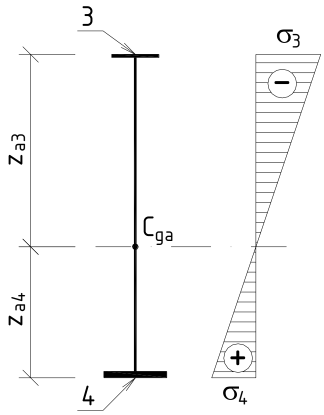


Fig. 10 Stress distribution in the steel cross-section

$$\sigma_{g1,3} = -\frac{M_{g1,k}}{I_{ya}} \cdot z_{a3} = -\frac{5\,048.0 \cdot 10^{-3}}{29.997 \cdot 10^{-3}} \cdot 1.225 = -\ 206.15 \text{ MPa}$$

$$\sigma_{g1,4} = \frac{M_{g1,k}}{I_{ya}} \cdot z_{a4} = \frac{5\,048.0 \cdot 10^{-3}}{29.997 \cdot 10^{-3}} \cdot 0.835 = \quad 140.52 \text{ MPa}$$

4.4.2 Stresses caused by part 2 of permanent loads (carried by composite beam)

4.4.2.1 Stresses at the time of loading (t_0 = 60 days)

$$\sigma_{g2,1} = -\frac{M_{g2,k}}{I_{y,ef(60)} \cdot n_0} \cdot z_{e1(60)} = -\frac{2\,408.0 \cdot 10^{-3}}{97.018 \cdot 10^{-3} \cdot 6.364} \cdot 0.493 = \ -1.92 \text{ MPa}$$

$$\sigma_{g2,2} = -\frac{M_{g2,k}}{I_{y,ef(60)} \cdot n_0} \cdot z_{e2(60)} = -\frac{2\,408.0 \cdot 10^{-3}}{97.018 \cdot 10^{-3} \cdot 6.364} \cdot 0.243 = -\ 0.95 \text{ MPa}$$

$$\sigma_{g2,3} = -\frac{M_{g2,k}}{I_{y,ef(60)}} \cdot z_{e3(60)} = -\frac{2\,408.0 \cdot 10^{-3}}{97.018 \cdot 10^{-3}} \cdot 0.243 = -\ 6.03 \text{ MPa}$$

$$\sigma_{g2,4} = \frac{M_{g2,k}}{I_{y,ef(60)}} \cdot z_{e4(60)} = \frac{2\,408.0 \cdot 10^{-3}}{97.018 \cdot 10^{-3}} \cdot 1.817 = \quad 45.10 \text{ MPa}$$

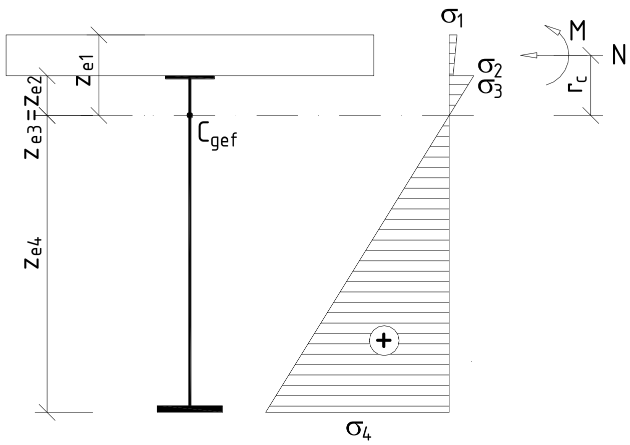


Fig. 11 Stress distribution in composite cross-section

4.4.2.2 Stresses at the time of introducing bridge into service (t = 120 days)

Gradual migration of normal stresses from the concrete slab to the steel beam due to creep of concrete may be considered by means of relaxation method, according to which the calculation is divided to two phases.

In the first phase it is assumed that after applying a long-term load (at the time t_0) the composite beam is fixated, in consequence of which the increase of deformation due to creep of concrete is zero. The creep of concrete causes that stresses in concrete slab gradually decrease – this phenomenon is denoted as relaxation. The relaxed value of stresses at the time of evaluation t may be calculated from expression

$$\sigma(t) = \sigma(t_0) \cdot \left(1 - \frac{\phi(t,t_0)}{1 + \psi_L \cdot \phi(t,t_0)}\right) = \sigma(t_0) \cdot \left(1 - \phi(t,t_0)\frac{n_0}{n_L}\right) = \sigma(t_0) \cdot \xi(t,t_0)$$

where $\xi(t,t_0)$ is denoted as relaxation factor. The initial stresses in the concrete slab $\sigma(t_0)$ may be replaced by corresponding normal force N and bending moment M (see Fig. 11):

$$N = \frac{\sigma_1 + \sigma_2}{2} \cdot A_c$$

$$M = \frac{\sigma_1 - \sigma_2}{2} \cdot \frac{I_c}{t_c/2} = \frac{\sigma_1 - \sigma_2}{t_c} \cdot I_c$$

Due to relaxation, these internal forces will decrease to

$$N_I = N \cdot \xi(t,t_0)$$
$$M_I = M \cdot \xi(t,t_0)$$

Then, the reactions in fictitious support are

$$N_{II} = N - N_I = N \cdot \left[1 - \xi(t,t_0)\right]$$
$$M_{II} = M - M_I = M \cdot \left[1 - \xi(t,t_0)\right]$$

In the second phase, after unlocking the fixation from the first phase, these reactions affect the composite cross-section, which is strained by eccentric compression and bending. The final stresses in the composite cross section are given by superposition of both steps of the relaxation method.

In our case, the effect of stresses $\sigma_{g2,1}$ and $\sigma_{g2,2}$ in the concrete slab at the time t_0 = 60 days may be replaced by normal force N and bending moment M

$$N = -\frac{(1.92 + 0.95) \cdot 10^3}{2} \cdot 0.850 = -1219.75 \text{ kN}$$

$$M = \frac{(1.92 - 0.95) \cdot 10^3}{0.25} \cdot 0.004427 = 17.18 \text{ kNm}$$

The relaxation factor at the time of introducing bridge into service (t = 120 days) is equal to

$$\xi(120,60) = 1 - \phi(120,60) \cdot \frac{n_0}{n_{L(120)}} = 1 - 0.62496 \cdot \frac{6.364}{10.738} = 0.6296$$

The stresses in concrete and corresponding internal forces decrease to following values

$$N_I = N \cdot \xi(120,60) = -1219.75 \cdot 0.6296 = -767.95 \text{ kN}$$

$$M_I = M \cdot \xi(120,60) = 17.18 \cdot 0.6296 = 10.82 \text{ kNm}$$

$$\sigma_{g2,1,I} = \sigma_{g2,1} \cdot \xi(120,60) = -1.92 \cdot 0.6296 = -1.21 \text{ MPa}$$

$$\sigma_{g2,2,I} = \sigma_{g2,2} \cdot \xi(120,60) = -0.95 \cdot 0.6296 = -0.60 \text{ MPa}$$

The reactions from fictitious support affecting the composite cross-section are

$$N_{II} = N \cdot \left[1 - \xi(120,60)\right] = -1219.75 \cdot (1 - 0.6296) = -451.80 \text{ kN}$$

$$M_{II} = M \cdot \left[1 - \xi(120,60)\right] = 17.18 \cdot (1 - 0.6296) = 6.36 \text{ kNm}$$

The moment due to eccentricity of the normal force N_{II} is

$$\Delta M_{II} = N_{II} \cdot r_c = 451.80 \cdot 0.523 = 236.29 \text{ kNm}$$

Corresponding stresses in the composite cross-section are as follows

$$\sigma_{g2,1,II} = \frac{1}{n_L} \cdot \left(-\frac{N_{II}}{A_{ef}} - \frac{M_{II} + \Delta M_{II}}{I_{y,ef}} \cdot z_{e1} \right) = \frac{1}{10.738} \cdot \left(-\frac{451.80}{129.156} - \frac{6.36 + 236.29}{86.274} \cdot 0.648 \right) = -0.50 \text{ MPa}$$

$$\sigma_{g2,2,II} = \frac{1}{n_L} \cdot \left(-\frac{N_{II}}{A_{ef}} - \frac{M_{II} + \Delta M_{II}}{I_{y,ef}} \cdot z_{e2} \right) = \frac{1}{10.738} \cdot \left(-\frac{451.80}{129.156} - \frac{6.36 + 236.29}{86.274} \cdot 0.398 \right) = -0.43 \text{ MPa}$$

$$\sigma_{g2,3,II} = -\frac{N_{II}}{A_{ef}} - \frac{M_{II} + \Delta M_{II}}{I_{y,ef}} \cdot z_{e3} = -\frac{451.80}{129.156} - \frac{6.36 + 236.29}{86.274} \cdot 0.398 = -4.62 \text{ MPa}$$

$$\sigma_{g2,4,II} = -\frac{N_{II}}{A_{ef}} + \frac{M_{II} + \Delta M_{II}}{I_{y,ef}} \cdot z_{e4} = -\frac{451.80}{129.156} + \frac{6.36 + 236.29}{86.274} \cdot 1.662 = 1.18 \text{ MPa}$$

Resultant stresses at the time of introducing bridge into service (t = 120 days) due to 2nd part of permanent loads including the creep effects are

$$\sigma_{g2,1} = \sigma_{g2,1,I} + \sigma_{g2,1,II} = -1.21 - 0.50 = -1.71 \text{ MPa}$$

$$\sigma_{g2,2} = \sigma_{g2,2,I} + \sigma_{g2,2,II} = -0.60 - 0.43 = -1.03 \text{ MPa}$$

$$\sigma_{g2,3} = \sigma_{g2,3(60)} + \sigma_{g2,2,II} = -6.03 - 4.62 = -10.65 \text{ MPa}$$

$$\sigma_{g2,4} = \sigma_{g2,4(60)} + \sigma_{g2,4,II} = 45.10 + 1.18 = 46.28 \text{ MPa}$$

4.4.2.3 Stresses at the end of service life (t = 36 525 days)

The relaxation factor at the end of service life (t = 36 525 days) is equal to

$$\xi(36\,525,60) = 1 - \phi(36\,525,60)\cdot\frac{n_0}{n_{L(36\,525)}} = 1 - 1.37771\cdot\frac{6.364}{16.008} = 0.4523$$

The stresses in concrete and corresponding internal forces decrease to following values

$$N_I = N\cdot\xi(36\,525,60) = -1219.75\cdot 0.4523 = -\,551.69\text{ kN}$$

$$M_I = M\cdot\xi(36\,525,60) = 17.18\cdot 0.4523 = 7.77\text{ kNm}$$

$$\sigma_{g2,1,I} = \sigma_{g2,1}\cdot\xi(36\,525,60) = -1.92\cdot 0.4523 = -\,0.87\text{ MPa}$$

$$\sigma_{g2,2,I} = \sigma_{g2,2}\cdot\xi(36\,525,60) = -0.95\cdot 0.4523 = -\,0.43\text{ MPa}$$

The reactions from fictitious support affecting the composite cross-section are

$$N_{II} = N\cdot\left[1-\xi(36\,525,60)\right] = -1219.75\cdot(1-0.4523) = -\,668.06\text{ kN}$$

$$M_{II} = M\cdot\left[1-\xi(36\,525,60)\right] = 17.18\cdot(1-0.4523) = 9.41\text{ kNm}$$

$$\Delta M_{II} = N_{II}\cdot r_c = 668.06\cdot 0.655 = 437.58\text{ kNm}$$

Corresponding stresses in the composite cross-section are as follows

$$\sigma_{g2,1,II} = \frac{1}{n_L}\cdot\left(-\frac{N_{II}}{A_{ef}} - \frac{M_{II}+\Delta M_{II}}{I_{y,ef}}\cdot z_{e1}\right) = \frac{1}{16.008}\cdot\left(-\frac{668.06}{103.10} - \frac{9.41+437.58}{77.220}\cdot 0.78\right) = -0.69\text{ MPa}$$

$$\sigma_{g2,2,II} = \frac{1}{n_L}\cdot\left(-\frac{N_{II}}{A_{ef}} - \frac{M_{II}+\Delta M_{II}}{I_{y,ef}}\cdot z_{e2}\right) = \frac{1}{16.008}\cdot\left(-\frac{668.06}{103.10} - \frac{9.41+437.58}{77.220}\cdot 0.53\right) = -0.60\text{ MPa}$$

$$\sigma_{g2,3,II} = -\frac{N_{II}}{A_{ef}} - \frac{M_{II}+\Delta M_{II}}{I_{y,ef}}\cdot z_{e3} = -\frac{668.06}{103.10} - \frac{9.41+437.58}{77.220}\cdot 0.53 = \quad -9.55\text{ MPa}$$

$$\sigma_{g2,4,II} = -\frac{N_{II}}{A_{ef}} + \frac{M_{II}+\Delta M_{II}}{I_{y,ef}}\cdot z_{e4} = -\frac{668.06}{103.10} + \frac{9.41+437.58}{77.220}\cdot 1.53 = \quad 2.38\text{ MPa}$$

Resultant stresses at the end of service life (t = 36 525 days) due to 2nd part of permanent loads including the creep effects are

$$\sigma_{g2,1} = \sigma_{g2,1,I} + \sigma_{g2,1,II} = \quad -\,0.87 - \; 0.69 = \quad -\,1.56\text{ MPa}$$

$$\sigma_{g2,2} = \sigma_{g2,2,I} + \sigma_{g2,2,II} = \quad -\,0.43 - \; 0.60 = \quad -\,1.03\text{ MPa}$$

$$\sigma_{g2,3} = \sigma_{g2,3(60)} + \sigma_{g2,2,II} = -\,6.03 - \; 9.55 = -\,15.58\text{ MPa}$$

$$\sigma_{g2,4} = \sigma_{g2,4(60)} + \sigma_{g2,4,II} = \; 45.10 + \; 2.38 = \quad 47{,}48\text{ MPa}$$

4.4.3 Variable traffic load

Normal stresses caused by the load group "gr1a" (LM1 + pedestrian load of footpaths) are independent on the time of evaluation, because variable load represents a short-time effect that is not affected by creep.

$$\sigma_{gr1a,1} = -\frac{M_{gr1a,k}}{I_{y,ef} \cdot n_0} \cdot z_{e1} = -\frac{7\,619.82 \cdot 10^{-3}}{97.018 \cdot 10^{-3} \cdot 6.364} \cdot 0.493 = \; -6.08 \text{ MPa}$$

$$\sigma_{gr1a,2} = -\frac{M_{gr1a,k}}{I_{y,ef} \cdot n_0} \cdot z_{e2} = -\frac{7\,619.82 \cdot 10^{-3}}{97.018 \cdot 10^{-3} \cdot 6.364} \cdot 0.243 = \; -3.00 \text{ MPa}$$

$$\sigma_{gr1a,3} = -\frac{M_{gr1a,k}}{I_{y,ef}} \cdot z_{e3} = -\frac{7\,619.82 \cdot 10^{-3}}{97.018 \cdot 10^{-3}} \cdot 0.243 = \; -19.09 \text{ MPa}$$

$$\sigma_{gr1a,4} = \frac{M_{gr1a,k}}{I_{y,ef}} \cdot z_{e4} = \frac{7\,619.82 \cdot 10^{-3}}{97.018 \cdot 10^{-3}} \cdot 1.817 = \; 142.71 \text{ MPa}$$

4.4.4 Stresses caused by shrinkage of concrete

Analogous to the creep effect, the effect of shrinkage of concrete may be calculated again in two steps. In the first step, it is assumed that the steel beam, restraining to free deformation of the concrete slab due to shrinkage, is absolutely rigid, in consequence of which there arise primary normal tensile stresses σ_{cs}, uniformly distributed past the depth of slab (Fig. 12). These stresses may be replaced by normal force $N_{cs} = \sigma_{cs} \cdot A_c$. In the second step, the reaction from fictitious support equal to this normal force affects the composite cross-section as an eccentric compression force. The final stresses in the composite cross section are given again by superposition of stresses obtained in the both steps.

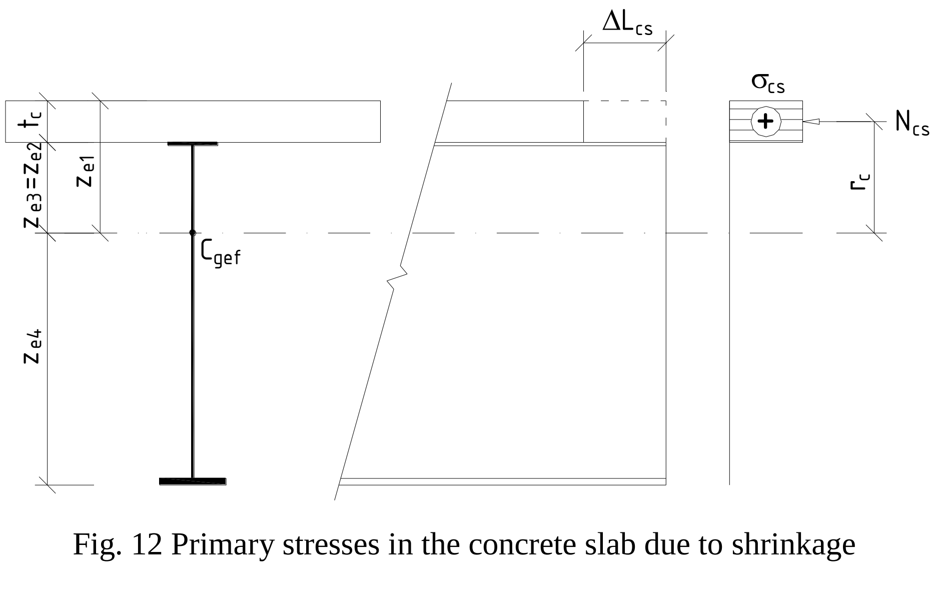


Fig. 12 Primary stresses in the concrete slab due to shrinkage

According to STN EN 1992-1-1, the total strain due to the shrinkage of concrete ε_{cs} is given by summation of two parts – the strain due to draining shrinkage ε_{cd} and the strain due to autogenous shrinkage $\varepsilon_{ca} \Rightarrow \varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca}$.

Development of the strain due to draining shrinkage ε_{cd} in time is defined by expression

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0}$$

where $k_h \cdot \varepsilon_{cd,0}$ is the final value of strain due to the drying shrinkage ($\varepsilon_{cd,\infty}$). The factor k_h (see Tab. 2) depends on equivalent depth of slab $h_0 = \frac{2A_c}{u}$ [mm], where A_c is area of the concrete slab and u is the perimeter of member that is in contact with atmosphere.

Tab. 2 Values of k_h

h_0	k_h
100	1.00
200	0.85
300	0.75
≥500	0.70

Coefficient $\beta_{ds}(t,t_s)$ describes development of drying shrinkage in time and may by calculated by using following expression

$$\beta_{ds}(t, t_s) = \frac{t - t_s}{(t - t_s) + 0.04 \cdot \sqrt{h_0^3}}$$

where t is an age of concrete [days] at the time of evaluation and t_s is an age of concrete at begin of the drying shrinkage, which should be generally assumed to be one day according to STN EN 1994-2.

The basic strain due to drying shrinkage may be obtained as follows

$$\varepsilon_{cd,0} = 0.85 \cdot \left[(220 + 110 \cdot \alpha_{ds1}) \cdot e^{\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cm0}} \right)} \right] \cdot 10^{-6} \cdot \beta_{RH}$$

$$\beta_{RH} = 1.55 \cdot \left[1 - (RH/100)^3 \right]$$

where $f_{cm} = f_{ck} + 8$ [MPa] is the mean compression strength of concrete at the age 28 days and $f_{cm0} = 10$ MPa.

The coefficients $\alpha_{ds1}, \alpha_{ds2}$ depend on the type of cement and are given by values:

- for cement class S: $\alpha_{ds1} = 3$, $\alpha_{ds2} = 0.13$
- for cement class N $\alpha_{ds1} = 4$, $\alpha_{ds2} = 0.12$
- for cement class R $\alpha_{ds1} = 6$, $\alpha_{ds3} = 0.11$

Development of the strain due to autogenous shrinkage ε_{ca} in time is defined by expression

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty)$$

where coefficient $\beta_{as}(t)$ describing development of autogenous shrinkage in time may by obtained from expression

$$\beta_{as}(t_s) = 1 - e^{-0.2\sqrt{t}}$$

and the final value of strain due to the autogenous shrinkage is given by expression

$$\varepsilon_{ca}(\infty) = 2.5 \cdot (f_{ck} - 10) \cdot 10^{-6}$$

In our case, on the assumption that the shrinkage will start to affect the composite girder from the first day after concreting (t_s = 1 day) and the cement class N will be applied, then at the relative humidity RH = 80% it will be valid:

$$\beta_{RH} = 1.55 \cdot \left[1 - (80/100)^3\right] = 0.7564$$

$$\varepsilon_{cd,0} = 0.85 \cdot \left[(220 + 110 \cdot 4) \cdot e^{\left(-0.12 \cdot \frac{38}{10}\right)}\right] \cdot 10^{-6} \cdot 0.7564 = 0.000269$$

$$\beta_{ds}(t, t_s) = \frac{120 - 1}{(120 - 1) + 0.04 \cdot \sqrt{250^3}} = 0.42943 \qquad \text{for } t = 120 \text{ days}$$

$$\beta_{ds}(t, t_s) = \frac{36\,525 - 1}{(36\,525 - 1) + 0.04 \cdot \sqrt{250^3}} = 0.99569 \qquad \text{for } t = 36\,525 \text{ days}$$

$$\varepsilon_{cd}(120) = 0.42943 \cdot 0.80 \cdot 0.000269 = \quad 0.0000924$$

$$\varepsilon_{cd}(36\,525) = 0.99569 \cdot 0.80 \cdot 0.000269 = \quad 0.0002143$$

$$\varepsilon_{ca}(\infty) = 2.5 \cdot (30 - 10) \cdot 10^{-6} = 0.00005$$

$$\beta_{as}(120) = 1 - e^{-0.2\sqrt{120}} = 0.88818$$

$$\beta_{as}(36\,525) = 1 - e^{-0.2\sqrt{36\,525}} = 1.00000$$

$$\varepsilon_{ca}(120) = 0.88818 \cdot 0.00005 = \quad 0.0000444$$

$$\varepsilon_{ca}(36\,525) = 1.00000 \cdot 0.00005 = \quad 0.0000500$$

The total strains due to shrinkage are:

- at the time t = 120 days: ε_{cs} = 0.0000924 + 0.0000444 = 0.0001368
- at the time t = 36 525 days: ε_{cs} = 0.0002143 + 0.0000500 = 0.0002643

4.4.4.1 Stresses at the time of introducing bridge into service (t = 120 days)

The primary normal stress in the concrete slab due to shrinkage is defined as follows:

$$\sigma_{cs} = \varepsilon_{cs}(t) \cdot \frac{E_{cm}}{1+\psi_L \cdot \phi_t} = \varepsilon_{cs}(t) \cdot E_{cm} \cdot \frac{n_0}{n_L} = 0.0001368 \cdot 33\,000 \cdot \frac{6.364}{12.032} = 2.39 \text{ MPa}$$

Corresponding normal force is

$N_{cs} = \sigma_{cs} \cdot A_c = 2.39 \cdot 10^3 \cdot 0.850 =$ 2031.50 kN

The moment due to eccentricity of the normal force N_{cs} is

$$M_{cs} = N_{cs} \cdot r_c = 2031.50 \cdot 0.560 = \ 1137.64 \text{ kNm}$$

The final stresses in the composite cross section due to shrinkage are

$$\sigma_{cs1} = \sigma_{cs} + \frac{1}{n_L} \cdot \left(-\frac{N_{cs}}{A_{ef}} - \frac{M_{cs}}{I_{y,ef}} \cdot z_{e1} \right) = 2.39 + \frac{1}{12.032} \cdot \left(-\frac{2031.50}{120.648} - \frac{1137.64}{83.741} \cdot 0.685 \right) = \ 0.22 \text{ MPa}$$

$$\sigma_{cs2} = \sigma_{cs} + \frac{1}{n_L} \cdot \left(-\frac{N_{cs}}{A_{ef}} - \frac{M_{cs}}{I_{y,ef}} \cdot z_{e2} \right) = 2.39 + \frac{1}{12.032} \cdot \left(-\frac{2031.50}{120.648} - \frac{1137.64}{83.741} \cdot 0.435 \right) = \ 0.50 \text{ MPa}$$

$$\sigma_{cs3} = -\frac{N_{cs}}{A_{ef}} - \frac{M_{cs}}{I_{y,ef}} \cdot z_{e3} = -\frac{2031.50}{120.648} - \frac{1137.64}{83.741} \cdot 0.435 = \ -22.75 \text{ MPa}$$

$$\sigma_{cs4} = -\frac{N_{cs}}{A_{ef}} + \frac{M_{cs}}{I_{y,ef}} \cdot z_{e4} = -\frac{2031.50}{120.648} + \frac{1137.64}{83.741} \cdot 1.625 = \ 5.24 \text{ MPa}$$

4.4.4.2 Stresses at the end of service life (t = 36 525 days)

The primary normal stress in the concrete slab due to shrinkage and corresponding internal forces affecting the composite cross-section are

$$\sigma_{cs} = \varepsilon_{cs}(t) \cdot E_{cm} \cdot \frac{n_0}{n_L} = 0.0002643 \cdot 33\,000 \cdot \frac{6.364}{16.744} = 3.31 \text{ MPa}$$

$N_{cs} = \sigma_{cs} \cdot A_c = 3.31 \cdot 10^3 \cdot 0.850 =$ 2813.50 kN

$$M_{cs} = N_{cs} \cdot r_c = 2813.50 \cdot 0.670 = \ 1855.05 \text{ kNm}$$

The final stresses in the composite cross section due to shrinkage are

$$\sigma_{cs1} = \sigma_{cs} + \frac{1}{n_L} \cdot \left(-\frac{N_{cs}}{A_{ef}} - \frac{M_{cs}}{I_{y,ef}} \cdot z_{e1} \right) = 3.31 + \frac{1}{16.744} \cdot \left(-\frac{2813.50}{100.765} - \frac{1855.05}{75.184} \cdot 0.795 \right) = \ 0.47 \text{ MPa}$$

$$\sigma_{cs2} = \sigma_{cs} + \frac{1}{n_L} \cdot \left(-\frac{N_{cs}}{A_{ef}} - \frac{M_{cs}}{I_{y,ef}} \cdot z_{e2} \right) = 3.31 + \frac{1}{16.744} \cdot \left(-\frac{2813.50}{100.765} - \frac{1855.05}{75.184} \cdot 0.545 \right) = \ 0.84 \text{ MPa}$$

$$\sigma_{cs3} = -\frac{N_{cs}}{A_{ef}} - \frac{M_{cs}}{I_{y,ef}} \cdot z_{e3} = -\frac{2813.50}{100.765} - \frac{1855.05}{75.184} \cdot 0.545 = \ -41.37 \text{ MPa}$$

$$\sigma_{cs4} = -\frac{N_{cs}}{A_{ef}} - \frac{M_{cs}}{I_{y,ef}} \cdot z_{e4} = -\frac{2813.50}{100.765} + \frac{1855.05}{75.184} \cdot 1.515 = \ 9.46 \text{ MPa}$$

4.5 Cross-section check

The final check of cross-section is presented in Tab. 3, where the values of normal stresses due to partial load effects are summarized. The variable loads are represented only by the load group "gr1a" and so no combination rules needs to be applied.

Tab. 3 Resultant normal stresses

Loads	Stresses due to partial loads [MPa]							
	At the time of introducing into service (t = 120 days)				At the end of service life (t = 36 525 days)			
	σ_1	σ_2	σ_3	σ_4	σ_1	σ_2	σ_3	σ_4
Part 1 of permanent loads	-	-	-206.15	140.52	-	-	-206.15	140.52
Part 2 of permanent loads	-1.71	-1.03	-10.65	46.28	-1.56	-1.03	-15.58	47.48
Shrinkage	0.22	0.50	-22.75	5.24	0.47	0.84	-41.37	9.46
Variable load - "gr1a"	-6.08	-3.00	-19.09	142.71	-6.08	-3.00	-19.09	142.71
Resultant stresses [MPa]	**-7.57**	**-3.53**	**-258.64**	**334.75**	**-7.17**	**-3.19**	**-282.19**	**340.17**
Design strength [MPa]	**30.00**	**30.00**	**355.00**	**355.00**	**30.00**	**30.00**	**355.00**	**355.00**

5 DESIGN OF SHEAR CONNECTION

Shear connection (together with transverse reinforcement) should be provided to transmit the longitudinal shear force between the concrete and the structural steel element, ignoring the effect of natural bond between the two. At present, headed stud shear connectors welded to the upper steel flange are mostly used. We propose the headed studs with diameter ϕ20 mm and overall nominal height 100 mm, made of steel with ultimate limit strength f_u = 340 MPa. The studs will be arranged in two lines.

Characteristic resistance of the stud is defined as the smaller value from:

$$P_{\mathrm{Rk}} = 0.8 \cdot f_{\mathrm{u}} \cdot \pi \cdot d^2/4 = 0.8 \cdot 340 \cdot 10^3 \cdot \pi \cdot 0.02^2/4 = 85.45 \text{ kN}$$

$$P_{\mathrm{Rk}} = 0.29 \cdot \alpha \cdot d^2 \cdot \sqrt{f_{ck} \cdot E_{cm}} = 0.29 \cdot 1.0 \cdot 0.02^2 \cdot \sqrt{30 \cdot 33000} = 230.84 \text{ kN}$$

where $\alpha = 0.2\left(\frac{h_{\mathrm{sc}}}{d} + 1\right)$ for $3 \le h_{\mathrm{sc}} / d \le 4$

$\alpha = 1$ for $h_{\mathrm{sc}} / d > 4$ (in our case $h_{\mathrm{sc}} / d = 100 / 20 > 4$)

It means that characteristic resistance of the stud is 85.45 kN. Then the design value is

$$P_{rd} = P_{rk} / \gamma_V = 85.45/1.25 = 68.36 \text{ kN}$$

The shear connectors should transmit the longitudinal force in the slab between the cross section with zero slip and the nearest cross section in which the maximum slip would occur in case of absence of shear connection (critical cross sections in Fig. 13). The necessary number of studs results from equilibrium condition

$n \cdot P_{rd} = N_c$

where N_c represents the maximum longitudinal force on the steel-concrete interface between the critical cross sections (mid-span and support), which is equal to normal resistance of the concrete slab.

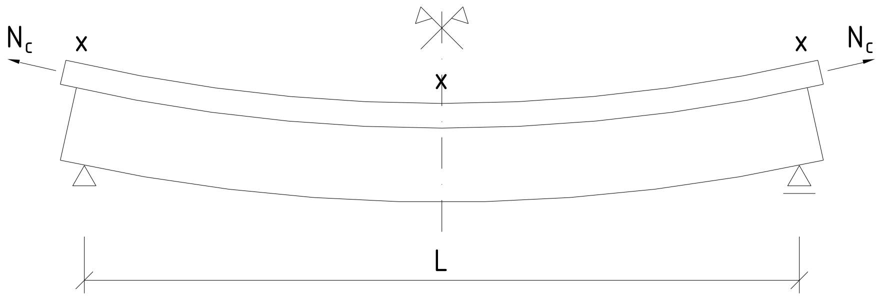


Fig. 13 Slip of the concrete slab in case of absence of shear connection
(x – critical cross sections)

$N_c = A_c \cdot f_{cd} = 0.85 \cdot 20 \cdot 10^3 = 17\,000$ kN

Then, the necessary theoretical number of studs (in the half of span) is:

$$n = \frac{N_c}{P_{rd}} = \frac{17\,000}{68.36} = 248.7$$

We propose 250 studs in two lines. Theoretical distances between the couples of studs are:

$$e = \frac{L/2}{n/2} = \frac{20\,000}{250/2} = 160 \text{ mm}$$

We propose following arrangement of studs in the half of span (Fig. 14):

71 × 140 mm + 53 × 190 mm (124 gaps or 125 couples of studs)

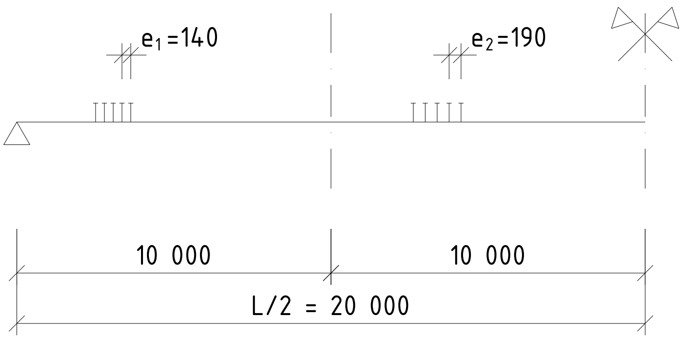


Fig. 14 Arrangement of studs on the beam

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